ECE 201 Spring 2010

Homework 13 Solutions

Problem 9

(a)

2R and 6R in series gives 8R. 8R and 8R in parallel gives 4R. Thus 12R is in series with V_s in the simplified circuit. Thus the Thevenin voltage is given by

$$\begin{aligned} V_{oc} &= \frac{V_s}{12R} \times \frac{1}{2} \times 6R \\ &= \frac{V_s}{4} \\ &= 30\ V \end{aligned}$$

To find the Thevenin resistance, we short circuit V_s. Thus 8R and 8R are in parallel, which gives 4R, which in turn is in series with 2R. This gives 6R and 6R in parallel. Thus $R_{th} = 3R = 900\ \Omega$.

(b)

Using the Thevenin equivalent circuit to simplify our calculations,

$$\begin{aligned} P_{R_L} &= \left(\frac{30}{900 + R_L}\right)^2 R_L \\ &= 0.1875\ W,\ 0.24\ W,\ 0.244898\ W\ (R_L = 300\ \Omega,\ 600\ \Omega,\ 1200\ \Omega) \end{aligned}$$

The Thevenin equivalent circuit analysis allows us to analyze the circuit without the load once and then plug in the various load resistor values to compute the relevant quantities. However, using earlier techniques would require 3 separate analyses to compute these values. Hence the use of a Thevenin equivalent reduces the effort needed to obtain the answers.

(c)

Using **Maximum Power Transfer Theorem**, for maximum power transfer, $R_L = 900\ \Omega$ and resultant power delivered to the load is given by $P_{max} = (30)^2/(4 \times 900) = 0.25\ W$.

(d)

If V_s is doubled, power becomes four times, as $P_{max} \propto V_s^2$. Thus power delivered to the load becomes $P_{load} = 0.25 \times 4 = 1\ W$.

Problem 17

We proceed to find the Thevenin voltage V_{oc} and Thevenin resistance R_{th}. Norton current is then given by $I_{sc} = V_{oc}/R_{th}$ and Norton resistance is same as Thevenin resistance. Since this problem involves a dependent source, we cannot apply the conventional method. We write KVL equations to find the required values.

$$\begin{aligned}
v_{AB} - 300i_A - v_x &= 0 \\
v_x - 200(i_A - v_x/18000) - \frac{4}{3}v_x &= 0 \\
\Rightarrow v_x &= -\frac{18000}{29}i_A \\
v_{AB} &= -320.689655i_A \\
v_{AB} &= R_{th}i_A + V_{oc} \\
\Rightarrow V_{oc} &= 0 \\
R_{th} &= -320.69\ \Omega \\
\Rightarrow I_{sc} &= 0 \\
R_{norton} &= -320.69\ \Omega
\end{aligned}$$

Problem 21

(a)

Again, we write KVL equations to find the Thevenin and Norton equivalent circuits.

$$\begin{aligned}
v_{AB} - 40i_A - 200(i_A + kv_x + v_x/50) &= 0 \\
V_s - v_x + 40i_A - v_{AB} &= 0
\end{aligned}$$

$$
\begin{aligned}
\Rightarrow v_{AB}\left[1+200(k+0.02)\right] &= i_A\left[240+8000(k+0.02)\right]+200V_s(k+0.02) \\
v_{AB} &= 60i_A+18 \\
v_{AB} &= i_A R_{th}+V_{oc} \\
\Rightarrow V_{oc} &= 18\ V \\
R_{th} &= 60\ \Omega \\
I_{sc} &= 0.3\ A
\end{aligned}
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(b)

Clearly, using the previous set of equations, V_{oc} is zero for $k=-0.02$ and for this value of k $R_{th}=240\ \Omega$.

Problem 37

(a)

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\begin{aligned}
v_{AB} &= i_A R_{th}+V_{oc} \\
54 &= 0.01R_{th}+V_{oc} \\
66 &= 0.04R_{th}+V_{oc} \\
\Rightarrow R_{th} &= 400\ \Omega \\
V_{oc} &= 50\ V \\
I_{sc} &= 0.125\ A
\end{aligned}
$$

(b)

$R_L=400\ \Omega,\ P_{max}=V_{oc}^2/4R_L=2500/(4\times 400)=1.5625\ W.$