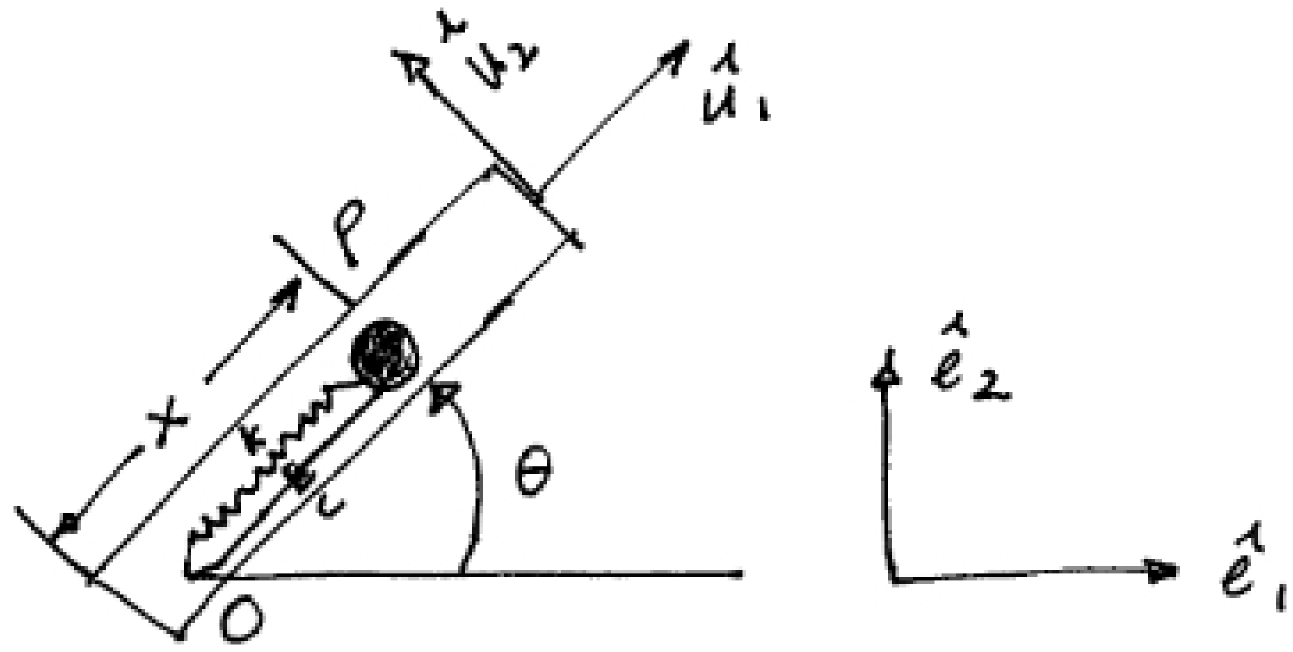
BRITTANY ESSINK
9/5/12

Updated by
Paul Witsberger
09/05/2017

1. MASS-SPRING-DAMPER IN A SPINNING TUBE



A PARTICLE, P, OF MASS m SLIDES IN A SMOOTH TUBE WHICH ROTATES AT A CONSTANT RATE, $d\theta/dt$. ASSUME THE UNSTRETCHED SPRING LENGTH, l_0 IS ZERO. THE SPRING HAS A CONSTANT OF k AND THE DAMPER HAS A DAMPING COEFICIENT OF c.

ASSUMPTIONS:

- NO FRICTION
- NO GRAVITY

EOM:

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{(k - m\dot{\theta}^2)}{m} x = 0$$

1a. LET $k = m(d\theta/dt)^2$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{(k - m\dot{\theta}^2)}{m} x = 0$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \cancel{\left(\frac{m\dot{\theta}^2 - m\dot{\theta}^2}{m}\right)x} = 0$$

CLASSICAL FORM OF EOM IS:

$$\ddot{x} + 2\zeta\omega_n\dot{x} = 0$$

PUT EQUATIONS INTO STATE VARIABLE FORM

$$y_1 = x \qquad \dot{y}_1 = \dot{x}$$
$$y_2 = \dot{x} \qquad \dot{y}_2 = \ddot{x}$$

THEREFORE

$$\dot{y}_1 = y_2$$
$$\dot{y}_2 = -2\zeta\omega_n y_2$$

GIVEN CONDITIONS:

$$x(0) = 0 \quad \dot{x}(0) = 1 \ \frac{\text{UNIT}}{\text{SECOND}} \implies \left(y_1(0) = 0,\ y_2(0) = 1\ \text{UNIT}/s\right)$$
$$\zeta = 0.5 \quad \omega_n = 1\ \text{RAD}/s$$

PLUGGING IN VALUES:

$$\dot{y}_1 = y_2$$
$$\dot{y}_2 = -y_2$$

$\boxed{\text{SEE ATTACHED FOR PLOT OF } x(t) \text{ VS. TIME}}$

1b. FOR THE SAME CONDITIONS IN 1a MAKE A PLOT OF $X_{numerical} - X_{analytical}$ TO SHOW THE DIFFERENCE BETWEEN THE NUMERICAL AND ANALYTICAL SOLUTION.

FROM HOMEWORK #2:

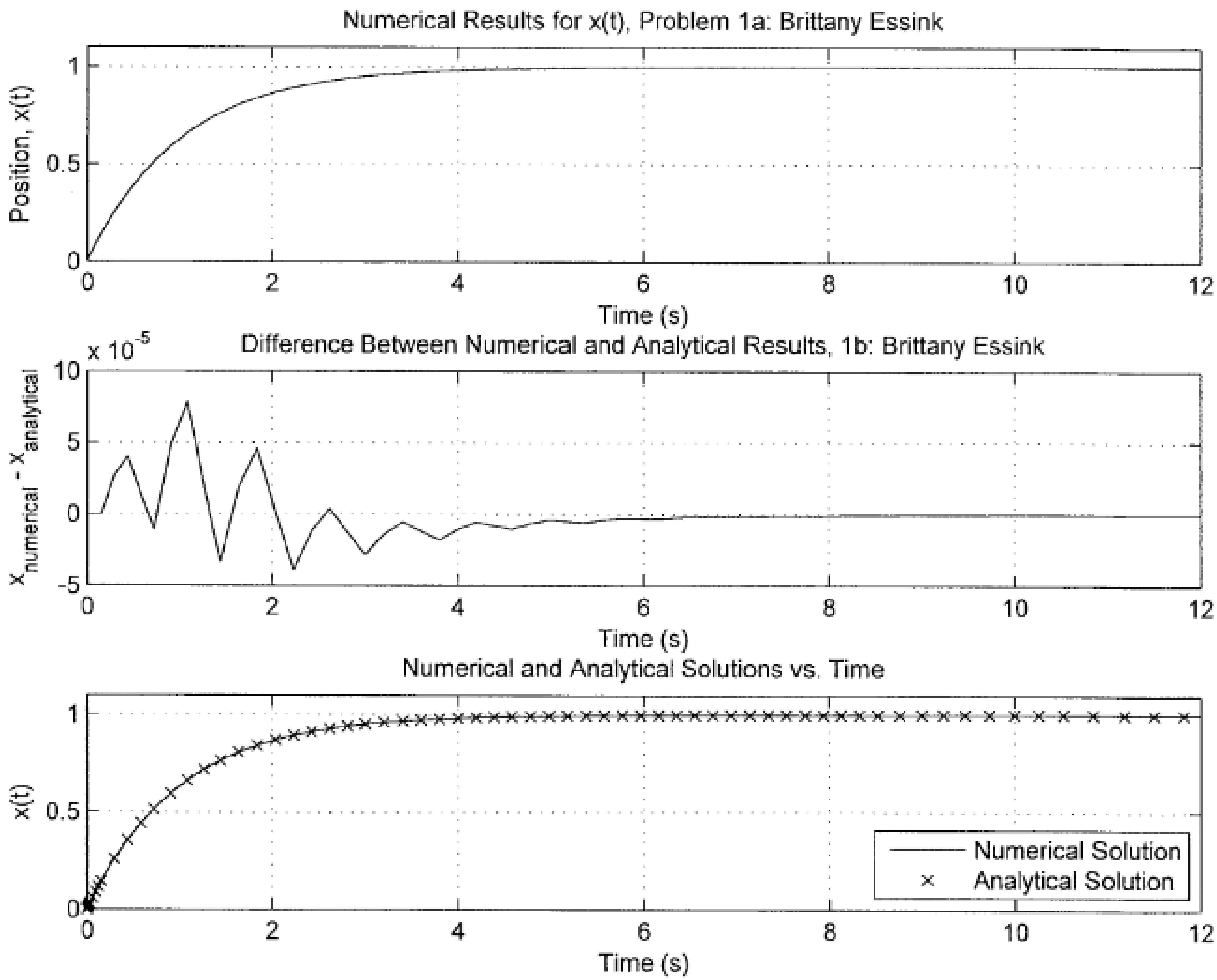
$$x(t) = x_0 + \frac{\dot{x}_0}{2\zeta\omega_n}\left(1 - e^{-2\zeta\omega_n t}\right)$$

AFTER PLUGGING IN INITIAL CONDITIONS, ζ, & ω_n:

$$x(t) = 1 - e^{-t}$$

$\boxed{\text{SEE ATTACHED FOR PLOT OF } (X_{numerical} - X_{analytical}) \text{ VS. TIME}}$

3


Numerical Results for x(t), Problem 1a: Brittany Essink
Position, x(t)
Time (s)
Difference Between Numerical and Analytical Results, 1b: Brittany Essink
x 10^-5
x_numerical - x_analytical
Time (s)
Numerical and Analytical Solutions vs. Time
x(t)
Numerical Solution
Analytical Solution
Time (s)