CSE310 HW03, Monday, 10/21/2013, Due: Monday, 10/28/2013

Please note that you have to typeset your assignment using either LaTeX or Microsoft Word. Hand-written assignment will not be graded. You need to submit a hardcopy before the lecture on the due date. You also need to submit an electronic version at the digital drop box. For the electronic version, you should name your file using the format CSE310-HW03-LastName-FirstName.

1. (10 pts) Let array A contain 10 elements from index 1 to index 10 as follows.

```
0, 1, 2, 3, 4, 5, 6, 7, 8, 9
```

In other words, $A[i] = i - 1$ for $i = 1, 2, \ldots, 10$. Suppose you are applying the linear time buildheap algorithm to build a **max-heap** from the 10 elements of array A. For $i = 5, 4, 3, 2, 1$, show the array structure after the call to `heapify(A, i)` is completed. You should specify **After heapify(A, 5), the array contents are:** followed by the array contents. Then do the same for $i = 4, 3, 2, 1$.

Answer:
After $heapify(A, 5)$, the array is

```
0, 1, 2, 3, 9, 5, 6, 7, 8, 4
```

After $heapify(A, 4)$, the array is

```
0, 1, 2, 8, 9, 5, 6, 7, 3, 4
```

After $heapify(A, 3)$, the array is

```
0, 1, 6, 8, 9, 5, 2, 7, 3, 4
```

After $heapify(A, 2)$, the array is

```
0, 9, 6, 8, 4, 5, 2, 7, 3, 1
```

After $heapify(A, 1)$, the array is

```
9, 8, 6, 7, 4, 5, 2, 0, 3, 1
```

Grading:

2 pts for each correct array.

2. (10 pts) Starting with an empty max-heap, you are to insert the numbers

   `0, 1, 2, 3, 4, 5, 6, 7, 8, 9`

   into the max-heap in the given order. Show the array contents after the first 6 insertions. Show the array contents after the first 7 insertions. Show the array contents after the first 8 insertions. Show the array contents after the first 9 insertions. Show the array contents after the first 10 insertions.

   Answer:

   After 6 iterations, the array is

   `5, 3, 4, 0, 2, 1`

   After 7 iterations, the array is

   `6, 3, 5, 0, 2, 1, 4`

   After 8 iterations, the array is

   `7, 6, 5, 3, 2, 1, 4, 0`

   After 9 iterations, the array is

   `8, 7, 5, 6, 2, 1, 4, 0, 3`

   After 10 iterations, the array is

   `9, 8, 5, 6, 7, 1, 4, 0, 3, 2`

   Grading:

   2 pts for each correct array.

3. (10 pts) In class, we have studied the linear time selection algorithm with group size 5. Suppose we will use group size 17, instead of 5. Again, we are using the median of medians as the pivot element. Assume that there are n elements, and all elements are distinct.

(4pts) What is the maximum number of elements that can appear on either side of the pivot element? You should not use the asymptotic notations.

Answer:

The maximum number of elements on either side is:
$n - 9\left\lceil \frac{\lceil n/17 \rceil}{2} - 2 \right\rceil \leq \frac{25}{34}n + 18.$

(4pts) Let $T(n)$ denote the worst-case time complexity of your algorithm. Write down the recurrence formula for $T(n)$.

Answer:

$T(n) = T(\frac{n}{17}) + T(\frac{25n}{34} + 18) + bn.$

(2pts) Use the asymptotic notations to denote $T(n)$ as a function of n.

Answer:

$T(n) = O(n)$

Grading:

Binary.

4. (10 pts) Suppose we use the first 8 natural numbers to denote 8 different objects. After some operations, the array contents are as follows:

```
-------------------------------
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
-------------------------------
| 5 |-1 | 7 | 8 |-1 | 2 |-1 |-1 |
-------------------------------
```

where the first row show the array indices, and the second row shows the corresponding array elements. Suppose we are using `union by rank` and `find without path compression`. Show the array contents after each of the following operations (these operations are applied in sequential order to the given disjoint-set).

(3pts) `union(1, 2)`

Answer:

```
-------------------------------
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
-------------------------------
| 5 |-2 | 7 | 8 | 2 | 2 |-1 |-1 |
```