

## CSE310 HW02 Grading Keys

Please note that you have to typeset your assignment using either  $\text{\LaTeX}$  or Microsoft Word. Hand-written assignment will not be graded. You need to submit a hardcopy before the lecture on the due date. You also need to submit an electronic version at the digital drop box. For the electronic version, you should name your file using the format CSE310-HW02-LastName-FirstName.

1. (10 pts) Algorithm  $A$  for solving problem  $P$  has a time complexity  $T_A(n)$  for an instance of size  $n$ . We do not know the asymptotic order of  $T_A(n)$ . However, we know that  $T_A(n) = 7 \cdot T_A(n/2) + 8n^2$ . Algorithm  $B$  for solving problem  $P$  has a time complexity  $T_B(n)$  for an instance of size  $n$ . We do not know the asymptotic order of  $T_B(n)$ . However, we know that  $T_B(n) = 8 \cdot T_B(n/2) + n^2$ .

Compute the asymptotic orders for  $T_A(n)$  and  $T_B(n)$  using the master method. You need to specify  $a$ ,  $b$ ,  $\log_b a$ , and decide the case. You also need to write the derived conclusion.

Which algorithm is asymptotically faster?

**Answer:**

For  $T_A(n)$ ,  $a = 7$ ,  $b = 2$ ,  $\log_b a = \log_2 7 \cong 2.8074$ . Choose  $\varepsilon = 0.5$ ,  $\log_b a - \varepsilon \cong 2.3074$ ,  $f(n) = O(n^{2.3074})$ . Thus  $T_A(n)$  belongs to case 1,  $T_A(n) = \Theta(n^{\log_2 7}) \cong \Theta(n^{2.8074})$ .

For  $T_B(n)$ ,  $a = 8$ ,  $b = 2$ ,  $\log_b a = 3$ . Choose  $\varepsilon = 1$ ,  $\log_b a - \varepsilon = 2$ ,  $f(n) = O(n^2)$ . Thus  $T_B(n)$  belongs to case 1,  $T_B(n) = \Theta(n^{\log_2 8}) = \Theta(n^3)$ .

Since  $n^{2.8074} < n^3$ ,  $T_A(n)$  is asymptotically faster than  $T_B(n)$ .

**Grading:**

4 pts for calculating  $T_A(n)$  (1 pt for  $a$ ,  $b$ , and  $\log_b a$ ; 2 pts for the case; 1 pt for the conclusion).

4 pts for calculating  $T_B(n)$  (1 pt for  $a$ ,  $b$ , and  $\log_b a$ ; 2 pts for the case; 1 pt for the conclusion).

2 pts for knowing that Algorithm A is faster than Algorithm B.

2. (10 pts)  $T(n) = 5 \cdot T(n/5) + 0.001n + \sqrt{n} \log_2 n$ . Use the master method to solve  $T(n)$ . You need to specify  $a$ ,  $b$ ,  $\log_b a$ , and decide the case. You also need to write the derived conclusion.

**Answer:**

$a = 5$ ,  $b = 5$ ,  $\log_b a = 1$ .

Since  $\lim_{n \rightarrow \infty} \frac{0.001n + \sqrt{n} \log_2 n}{n} = 0.001 + \lim_{n \rightarrow \infty} \frac{\log_2 n}{\sqrt{n}} = 0.001$ ,  $f(n) = \Theta(n)$ . Thus,  $T(n)$  belongs to case 2.

Therefore,  $T(n) = \Theta(n^{\log_b a} \lg n) = \Theta(n \lg n)$ .

**Grading:**

3 pts for correctly specifying  $a$ ,  $b$ , and  $\log_b a$ .

4 pts for deciding the right case.

3 pts for calculating  $T(n)$  correctly.

3. (10 pts)  $T(n) = 7 \cdot T(n/2) + n^{2.85}$ . Use the master method to solve  $T(n)$ . You need to specify  $a$ ,  $b$ ,  $\log_b a$ , and decide the case. You also need to write the derived conclusion.

**Answer:**

$a = 7$ ,  $b = 2$ ,  $\log_b a = \log_2 7 \cong 2.8074$ . Choose  $\varepsilon = 0.04$ ,  $\log_b a + \varepsilon \cong 2.8474$ .  $f(n) = n^{2.85} = \Omega(n^{2.8474})$ .

$a \cdot f(n/b) = 7(n/2)^{2.85} = \frac{7}{2^{2.85}} n^{2.85} \cong 0.9709 n^{2.85}$ , Choose  $c = 0.98 < 1$ , then  $a \cdot f(n/b) \leq c \cdot f(n)$  for  $\forall n \geq 0$ . Thus,  $T(n)$  belongs to case 3.

Therefore,  $T(n) = \Theta(f(n)) = \Theta(n^{2.85})$ .

**Grading:**

3 pts for correctly specifying  $a$ ,  $b$ , and  $\log_b a$ .

4 pts for deciding the right case.

3 pts for calculating  $T(n)$  correctly.

4. (10 pts)  $T(n) = 3 \cdot T(n/3) + n^2$ . Use the master method to solve  $T(n)$ . You need to specify  $a$ ,  $b$ ,  $\log_b a$ , and decide the case. You also need to write the derived conclusion.

**Answer:**

$a = 3$ ,  $b = 3$ ,  $\log_b a = 1$ . Choose  $\varepsilon = 1$ ,  $\log_b a + \varepsilon = 2$ .  $f(n) = n^2 = \Omega(n^{\log_b a + \varepsilon})$ .

$a \cdot f(n/b) = \frac{3}{9} n^2 \cong 0.3333 n^2$ . Choose  $c = 0.5 < 1$ , then  $a \cdot f(n/b) \leq c \cdot f(n)$  for  $\forall n \geq 0$ . Thus,  $T(n)$  belongs to case 3.

Therefore,  $T(n) = \Theta(f(n)) = \Theta(n^2)$ .

**Grading:**

3 pts for specifying  $a$ ,  $b$ , and  $\log_b a$ .

4 pts for deciding the right case.

3 pts for calculating  $T(n)$  correctly.

5. (10 pts) Given the sequence

$A[1]=50$ ,  $A[2]=40$ ,  $A[3]=30$ ,  $A[4]=20$ ,  $A[5]=10$ .

List all of the inversions in the sequence.

**Answer:**

$\langle 1, 2 \rangle, \langle 1, 3 \rangle, \langle 1, 4 \rangle, \langle 1, 5 \rangle, \langle 2, 3 \rangle, \langle 2, 4 \rangle, \langle 2, 5 \rangle, \langle 3, 4 \rangle, \langle 3, 5 \rangle, \langle 4, 5 \rangle.$

**Grading:**

1 pt for each inversion.