Test Total

Name _____

Test 3 Honors Calculus III 3450:223:005 Dr. Norfolk November 13, 2003

Show all of your work and explain your reasoning.

1. Find a function $f(x, y)$ and a region R such that

$$\int\int_R f(x,y)\ dA = \lim_{m,n\to\infty} \sum_{i=1}^{m}\sum_{j=1}^{n} \left[\left(2+\frac{3i}{m}\right)\left(1+\frac{2j}{n}\right) - \cos^2\left(2+\frac{3i}{m}\right)\right].\frac{3}{m}.\frac{2}{n}$$

5 points

2. Evaluate $\int_0^2 \int_1^4 x^2 e^{-y}\ dxdy$

10 points

3. Consider $\int_0^1 \int_x^{2x} x - y^2\ dydx$.

   (a) Evaluate this integral.

15 points

   (b) Sketch the region of integration.

5 points

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(c) SET UP, BUT DO NOT EVALUATE, the problem with the order of integration reversed.

5 points

4. We may write

$$\left(\int_{-\infty}^{\infty} e^{-x^2}\,dx\right)^2 = \int_{-\infty}^{\infty} e^{-x^2}\,dx \int_{-\infty}^{\infty} e^{-y^2}\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2}\,dA = \lim_{R\to\infty} \int\int_{x^2+y^2\le R^2} e^{-x^2-y^2}\,dA$$

By evaluating the last quantity using polar coordinates, find the value of $\int_{-\infty}^{\infty} e^{-x^2}\,dx$.

15 points

5. A lamina is bounded by the circle $x^2+y^2=1$, $x, y \ge 0$, and has density $\rho(x,y) = x+2y+1$. WITHOUT EVALUATING THE INTEGRALS, find a formula using iterated integrals for the x-coordinate of the centre of mass.

10 points

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6. WRITE, BUT DO NOT EVALUATE, an iterated integral for the surface area of the paraboloid $z = 9 - x^2 - 3y^2$ which lies above the triangle in the xy-plane with vertices (0,0), (4,0) and (4,2).

10 points

7. A tetrahedron is bounded by $x+4y+6z = 12$, $x, y, z \geq 0$, and has density function $\rho(x, y, z) = (x + y)e^{1-2z}$.

   WRITE, BUT DO NOT EVALUATE, an iterated triple integral for the mass of the object.

10 points

8. The region D is bounded by the cylinder $x^2 + y^2 = 9$, $z = 0$, $z = 4$, $y = 0$, $y = x$, $x \geq 0$. REWRITE, BUT DO NOT EVALUATE a simplified integral in cylindrical coordinates for $\iiint_D x^2 dV$.

10 points

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