251y0322 10/27/03

ECO251 QBA1
SECOND HOUR EXAM
March 21, 2003

Name: ____KEY_____
Social Security Number: _____

Part I: (48 points) Do all the following: All questions are 2 points each except as marked. Exam is normed on 50 points including take-home. **Please re-read, 'Things that You should never do on an Exam or Anywhere Else, '** and especially recall that a probability cannot be above 1!

The following joint probability table shows the relation between two sets of events. Let event A be that the individual is below 22 ($\overline{A}$ is over 21), and the event B_0 be that the individual had no traffic violations in the last 18 months, the event B_1 be that the individual has one traffic violation in the last 18 months and the event B_2 be that the individual has 2 traffic violations over the last 18 months. No individuals in this pool of drivers has more than 2 violations.

	B_0	B_1	B_2
A	.22	.12	.06
$\overline{A}$	.41	.18	.01

Note, to do the problems below, at least total the rows and columns -

	B_0	B_1	B_2	
A	.22	.12	.06	.40
$\overline{A}$	.41	.18	.01	.60
	.63	.30	.07	1.00

1. The probability that someone who is over 21 has no traffic violations is (To 2 decimal places):
   a) .63
   b) .60.
   c) .41
   d) *.68 You have been asked for $P(B_0|\overline{A}) = \frac{P(B_0 \cap \overline{A})}{P(\overline{A})} = \frac{.41}{.60} = .683$
   e) None of the above.

2. The probability that someone picked at random is over 21 and has no traffic violations is (To 2 decimal places):
   a) .63
   b) .60.
   c) *.41 Joint probabilities are what the table shows!
   d) .68
   e) None of the above.

3. The probability that someone chosen at random is either under 22 or has 2 violations is:
   a) .40
   b) .06
   c) .46
   d) .47
   e) *.41 $P(A \cup B_2) = P(A) + P(B_2) - P(A \cap B_2) = .40 + .07 - .06 = .41$
   f) None of the above

	B_0	B_1	B_2	
A	.22	.12	.06	.40
$\overline{A}$	.41	.18	.01	.60
	.63	.30	.07	1.00

4. Which two events are independent?
   a) A and $\overline{A}$ Note that 'mutually exclusive' and 'independent are almost opposites.
   b) $\overline{A}$ and B_2
   c) * $\overline{A}$ and B_1. The definition of independence is $P(\overline{A}|B_1) = P(\overline{A})$. In this case $P(\overline{A}) = .60$ and $P(\overline{A}|B_1) = \frac{P(\overline{A} \cap B_1)}{P(B_1)} = \frac{.18}{.30} = .60$. But a better way to do this is to note that $P(\overline{A} \cap B_1) = P(\overline{A})P(B_1) = .60(.30) = .18$.
   d) A and B_0
   e) A and B_2
   f) None of these.

5. Which two events are mutually exclusive?
   a) * A and $\overline{A}$ Complements are always mutually exclusive. None of the other pairs have a joint probability of zero.
   b) $\overline{A}$ and B_2
   c) $\overline{A}$ and B_1.
   d) A and B_0
   e) A and B_2
   f) None of these.

In questions 6 and 7 you need to know what $P(B_0)$, $P(B_1)$ and $P(B_2)$ are to do the problems. Show your work.

Event	$P(x)$	$xP(x)$	$x^2P(x)$
$x = 0$	.63	0	0
$x = 1$	.30	0.30	0.30
$x = 2$	.07	0.14	0.28
Total	1.00	0.44	0.58

Solution: We can use the following table.

6. What is the probability that a person picked at random has at least one violation?
   Solution: $P(x \geq 1) = P(1) + P(2) = .30 + .07 = .37$ 'For the 200th time, 'at least one' and 'exactly one' are rarely the same thing.

7. What is the mean and the standard deviation of the number of violations our drivers have? (6) 18
   Solution: From the work above $\mu = \sum xP(x) = 0.44$.
   $\sigma^2 = E(x^2) - \mu^2 = .58 - (.44)^2 = .3864$
   $\sigma = \sqrt{.3864} = 0.6216$ This can't be $\overline{x}$ and S. These are a sample mean and variance, but there is no sample.

8. Which of the following statements must be true if $P(A)=.6$, $P(B)=.4$ and $P(A\cap B)=0$

(i) A and B are mutually exclusive. **Explanation:** $P(A\cap B)=0$

(ii) A and B are independent. **Explanation:** $P(A\cap B)=P(A)(P(B))$

(iii) A and B are collectively exhaustive. **Explanation:** $P(A\cup B)=1$

(iv) $P(A|B)=0$ **Explanation:**

$P(A|B)=P(A\cap B)/P(B)$

a) All are true but (i)
b) *All are true but (ii)
c) All are true but (iii)
d) All are true but (iv)
e) All are true
f) None are true.

9. According to a survey of American households, the probability that the residents own 2 cars if annual household income is over $25,000 is 80%. Of the households surveyed, 60% had incomes over $25,000 and 70% had 2 cars. The probability that the residents of a household own 2 cars and have an income less than or equal to $25,000 a year is (3):

a) 0.12.
b) 0.18.
c) *0.22.
d) 0.48.

Solution: The easiest way I know is to start with the given facts. Let 'two' be the event that a household owns two cars, and '$25+$' be the event that a family has an income over $25000. The problem says $P(two|25+)=.80$, $P(25+)=.60$, $P(two)=.70$ and asks for $P(two\cap\overline{25+})$

Using the second two facts we get

	$25+$	$\overline{25+}$	
two	__	__	.70
$\overline{two}$	__	__	
	.60	__	1.00

. But 80% of the 60% of families that own two cars have incomes over 25000, that is $P(two\cap 25+)=P(two|25+)P(25+)=.80(.60)=.48$. So now we have

	$25+$	$\overline{25+}$	
two	.48	__	.70
$\overline{two}$	__	__	
	.60	__	1.00

If we fill in more, we get