

ECO251 QBA1
THIRD EXAM
Apr 24, 2008
TAKE HOME SECTION

Name: _____
Student Number: _____

Throughout this exam **show your work!** Please indicate clearly what sections of the problem you are answering and what formulas you are using. **Write on one side of the page!**

Part III. Do all the Following (25+ Points) **Show your work! Neatness counts! Answers of 'zero' or 'one' especially are unacceptable without an explanation. Do not use one distribution to approximate another without justifying the replacement!**

1. Identify the distribution that you are using in each problem. If you have a number like $n = 20 + g$, make it very clear what value of n you are using. **Look at the solved problems for Section L, the solution to Grass3 and 'Great Distributions' (especially the hints on the 3rd page) before you start.**

Let g be the last digit of your student number. If $g = 0$, change it to 2. Hint: It may help in these problems to realize that if $n = 100$, 93 or more is 93 to 100 and 10 or less is 10 to zero. This is especially useful if we count failures.

a. A basketball player makes $(50 + 5g)\%$ of her free throws over the basketball season. In one game she gets 20 free throws and misses $(13 - g)$ of them. The coach will investigate if the probability of doing as badly as she did or worse is below 5%. Will the coach investigate? Do the math to justify your answer. (2)

b. We believe that 80% of all accidents involve alcohol. If we investigate $(6 + g)$ accidents, what is the probability that $(2 + g)$ or fewer involve alcohol? (1)

c. A test consists of $6 + g$ questions and a student must get at least 70% of the questions right to pass the course. Each question is a multiple choice question with 2 possible answers. Note that if $n = 16$, 70% of 16 is 11.2, so 12 or more must be right. What is the probability of passing the exam if you just guess? What happens to the probability as the size of the exam increases to 20 questions? (2)

Assume that the professor instead offers an exam with $6 + g$ questions with four choices and then says that a passing grade will be 50% or more. Does this raise or lower the chance of passing for the guesser? What happens to this probability as the size of the exam rises to 20? (2)

d. (Dummeldinger) The diameter of ball bearings has a continuous uniform distribution over the interval $3 + .1g$ millimeters to $5 - .1g$ millimeters.

(i) If ball bearings are only acceptable if they are between 4 and 5 millimeters, what proportion of those produced is defective?

(ii) If I take a sample of three ball bearings from the assembly line.

What is the chance that none of the ball bearings are defective?

(iii) What is the chance that exactly one is defective?

(iv) What is the chance that at least one is defective?

(v) What is the chance that the third ball bearing that I inspect is the first one that is defective?

[12]

e. Supposedly 60% of drivers wear their seatbelts. 1000 drivers are stopped.

(i) What is the approximate probability that less than half were wearing seatbelts? (1)

(ii) What is the approximate probability that less than $(200 + g)$ were wearing seatbelts? (1)[14]

f. If each member of the State Highway patrol writes an average of 10 tickets a day,

(i) What is the chance that a given policeman will write more than $(10 + g)$ tickets in a day? (1)

- (ii) Assuming that a policeman averages 50 tickets during a work week, what is the chance that a given policeman will write more than $5(10 + g)$ tickets in a week? (I strongly doubt that the answer is the same as the answer to (i)) (2)
- g. A 'psychic' is shown 5 cards and then is expected to be able to guess which one of the cards is randomly drawn. If the 'psychic' is a fake and is merely randomly guessing what card is picked and the experiment is repeated 20 times, what is the chance that the psychic will pick the right card $(4 + g)$ or more times? (1)
- h. The average number of small business bankruptcies in Fredonia are $(13 + .1g)$ a month. What is the probability of at least one bankruptcy in a month? What is the probability of at least 3 in a month? What is the probability of more than 180 in a year? (3) [21]
- i. There are exactly 14 firms in the US that produce jorcillators and $(1 + g)$ produce computers as well. If you pick a random sample of 3 firms to examine, what is the chance that at least one will be a computer producer. What is the chance that all the firms in your sample are computer producers? Assume that the same proportion of 70 companies produce computers what would the probability that at least one and all three in a 3-firm sample be computer producers? (4) [25]
- j. (Extra credit) The time between landings at an airport follows an exponential distribution with a mean of 30 seconds. What is the probability that there will be a gap of between $(15 + g)$ and $(45 + g)$ seconds before the next plane lands?

2. As everyone knows, a jorcillator has two components, a phillinx and a flubberall. It seems that the jorcillator only works as long as either component works (so that it fails in the first period only if both components fail). **Note**

The probability of the phillinx failing is given by a Normal distribution with $\mu = 18 + g$ and $\sigma = 3$. For example, if the life of the phillinx is represented by x_1 , the chance of the phillinx failing in the first ten years is $P(0 \leq x_1 \leq 10)$ and the probability of it failing in the second ten years is $P(10 \leq x_1 \leq 20)$. For example: Ima Badrisk has the number 375292, so her distribution has a mean of $\mu = 18 + .g = 20$. The flubberall has exactly the same distribution.

In order to maintain my sanity, use the following events. Period 1 is the first ten years, period 2 is the second ten years and period three is happily ever after.

Failure of the phillinx in period 1, 2, 3 are events A_1 , A_2 and A_3

Failure of the flubberall in period 1, 2, 3 are events B_1 , B_2 and B_3

Failure of the jorcillator in period 1, 2, 3 are events C_1 , C_2 and C_3 .

- What is the probability that the phillinx will fail in period 1? Period 2? Period 3? (2)
- What is the probability that the jorcillator will fail in the first period? (1)
- What is the probability that the jorcillator will fail in the second period? (1)
- What is the probability that the jorcillator will fail in the third period? (1)

If you haven't figured it out already, one of the easiest ways to do this is to make a joint probability table. Put the A events across the top. Put the B events down the side. Figure out what the probability of the joint events must be if they are independent. Now make a similar table. This time, instead of probabilities, fill in the period in which the jorcillator fails.

- (Extra Credit) Find the probability that the jorcillator and the Phillinx both fail in the third **period** (1)
- (Extra Credit) Find the probability that the jorcillator fails in the third **period**, given that the phillinx fails in the third month i.e. $P(C_3|A_3)$ (1)
- Demonstrate Bayes' rule by finding $P(A_3|C_3)$ and showing that Bayes' rule explains the relationship between the conditional probabilities that you have found. (2) [34]

3. Assume that you have a variable with a Poisson distribution with a mean of $(1.5 - .1g)$. (My value of g is 2, so I have a mean of $(1.5 - .1(2)) = 1.5 - .2 = 1.3$). Write down the values of $P(0)$, $P(1)$ etc for all values of X that have probabilities of .0001 or larger by subtracting values from the Poisson table. Compute the expected value and standard deviation using the probabilities and values of X and show that these are the same as stated in our write-ups on the Poisson distribution.
(4) [37]