SECOND PRACTICE EXAM SECOND HOURLY **Math 21b, Fall 2003**

Name:

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- Start by writing your name in the above box and check your section in the box to the left.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) (20 points) True or False? No justifications are needed.

T	F	If A is a non-invertible $n \times n$ matrix, then $\det(A) \neq \det(\mathrm{rref}(A))$.

T	F	If the rows of a square matrix form an orthonormal basis, then the columns must also form an orthonormal basis.
T	F	If the diagonal entries of an $n \times n$ matrix A are odd integers and all the entries not lying on the diagonal are even integers, then A is invertible.
T	F	A 2×2 rotation matrix $A \neq I_2$ does not have any real eigenvalues.
T	F	If A and B both have $\vec{v}$ as an eigenvector, then $\vec{v}$ is an eigenvector of AB.
T	F	If A and B both have λ as an eigenvalue, then λ is an eigenvalue of AB.
T	F	Similar matrices have the same eigenvectors.
T	F	If a 3×3 matrix A has 3 independent eigenvectors, then A is similar to a diagonal matrix.
T	F	If a square matrix A has non-trivial kernel, then 0 is an eigenvalue of A.
T	F	If the rank of an $n \times n$ matrix A is less than n, then 0 is an eigenvalue of A.
T	F	Two diagonalizable matrices whose eigenvalues are equal must be similar.
T	F	A square matrix A is diagonalizable if and only is A^2 is diagonalizable.
T	F	If a square matrix A is diagonalizable, then $(A^T)^2$ is diagonalizable.
T	F	If a square matrix A has k distinct eigenvalues, then $\text{rank}(A) \geq (k-1)$.
T	F	There exist matrices A with k distinct eigenvalues whose rank is strictly less than k.
T	F	If A is an $n \times n$ matrix which satisfies $A^k = 0$ for some positive integer k, then all the eigenvalues of A are 0.
T	F	If a 3×3 matrix A satisfies $A^2 = I_3$ and A is diagonalizable, then A must be similar to the identity matrix.
T	F	A and A^T have the same eigenvectors.
T	F	If A and B are diagonalizable, AB is also diagonalizable.
T	F	The least squares solution of a system $A\vec{x} = \vec{b}$ is unique if and only if $\ker(A) = 0$.

Problem 3) (10 points)

Find the volume of the three dimensional parallelepiped in four dimensions which is spanned by the vectors $\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$.

Problem 4) (10 points)

Assume that A is a skew-symmetric matrix, that is, it is a $n \times n$ matrix which satisfies $A^T = -A$.

a) Find $\det(A)$ if n is odd.

b) What possible values can $\det(A)$ have if n is even?

c) Verify that if λ is an eigenvalue of A, then $-\lambda$ is also an eigenvalue of A.

Problem 5) (10 points)

The recursion

$$u_{n+1} = u_n - u_{n-1} + u_{n-2}$$

is equivalent to the discrete dynamical system

$$\begin{bmatrix} u_{n+1} \\ u_n \\ u_{n-1} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u_n \\ u_{n-1} \\ u_{n-2} \end{bmatrix} = A \begin{bmatrix} u_n \\ u_{n-1} \\ u_{n-2} \end{bmatrix}.$$

a) Find the (real or complex) eigenvalues of A.

b) Is there a vector $\vec{v}$ such that $||A^n \vec{v}|| \to \infty$?

c) Can you find any positive integer k such that $A^k = I_3$?

Problem 6) (10 points)

Let A be the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

a) Find $\det(A)$.

b) Find all eigenvalues whether real or complex of A and state their algebraic multiplicities.

c) For each real eigenvalue λ of A find the eigenspace and the geometric multiplicity.

Problem 7) (10 points)

Find S and a diagonal matrix B such that $S^{-1}AS = B$, where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{bmatrix}.$$

Problem 8) (10 points)

Find the function of the form

$$f(t) = a\sin(t) + b\cos(t) + c$$