The 90° Hybrid Coupler

The 90° Hybrid Coupler is a 4-port device, otherwise known as the **quadrature** coupler or **branch-line** coupler. Its scattering matrix (ideally) has the **symmetric** solution for a matched, lossless, reciprocal 4-port device:

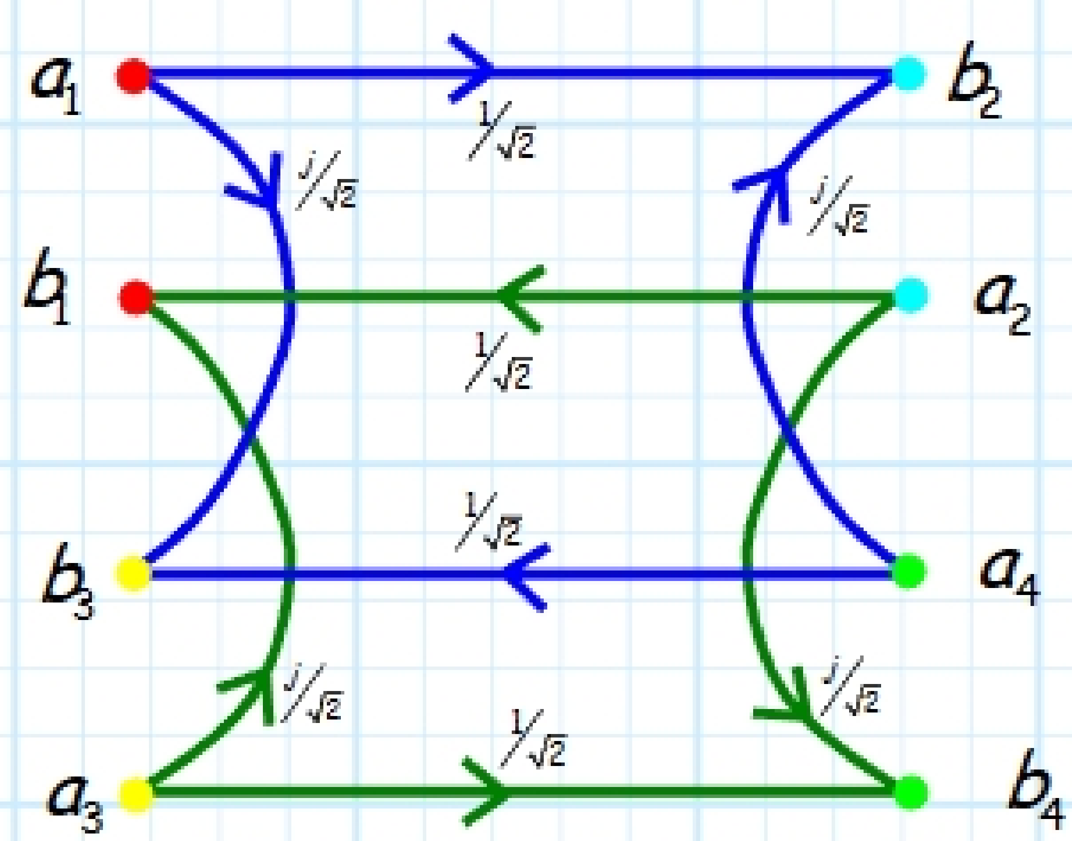
$$\mathcal{S} = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

However, for **this** coupler we find that

$$\alpha = \frac{-j}{\sqrt{2}} \qquad\qquad j\beta = \frac{-1}{\sqrt{2}}$$

Therefore, the **scattering matrix** of a quadrature coupler is:

$$\mathcal{S} = \begin{bmatrix} 0 & -j/\sqrt{2} & -1/\sqrt{2} & 0 \\ -j/\sqrt{2} & 0 & 0 & -1/\sqrt{2} \\ -1/\sqrt{2} & 0 & 0 & -j/\sqrt{2} \\ 0 & -1/\sqrt{2} & -j/\sqrt{2} & 0 \end{bmatrix}$$



It is evident that, just as with the directional coupler, the ports are **matched** and the device is **lossless**. Note also, that if a signal is incident on one port only, then there will be a port from which **no** power will exit (i.e., an **isolation** port).

Unlike the directional coupler, the power that is flows into the input port will be **evenly** divided between the two non-isolated ports.

For example, if 10 mW is incident on port 3 (and all other ports are matched), then 5 mW will flow out of **both** port 1 and port 4, while no power will exit port 2 (the isolated port).

Note however, that the although the **magnitudes** of the signals leaving ports 1 and 4 are **equal**, the relative **phase** of the two signals are separated by **90 degrees** ($e^{j\pi/2} = j$).

We find, therefore, that if in **real** terms the voltage out of port 1 is:

$$v_1(z,t) = \frac{|V_{03}^-|}{\sqrt{2}} \cos(\omega_0 t + \beta z)$$

then the signal form port 4 will be:

$$v_4(z,t) = \frac{|V_{03}^-|}{\sqrt{2}} \sin(\omega_0 t + \beta z)$$

There are **many** useful applications where we require both the **sine** and **cosine** of a signal!

Q: *But how do we **construct** this device?*

A: Similar to the Wilkinson power divider, we construct a quadrature hybrid with **quarter-wavelength** sections of transmission lines.

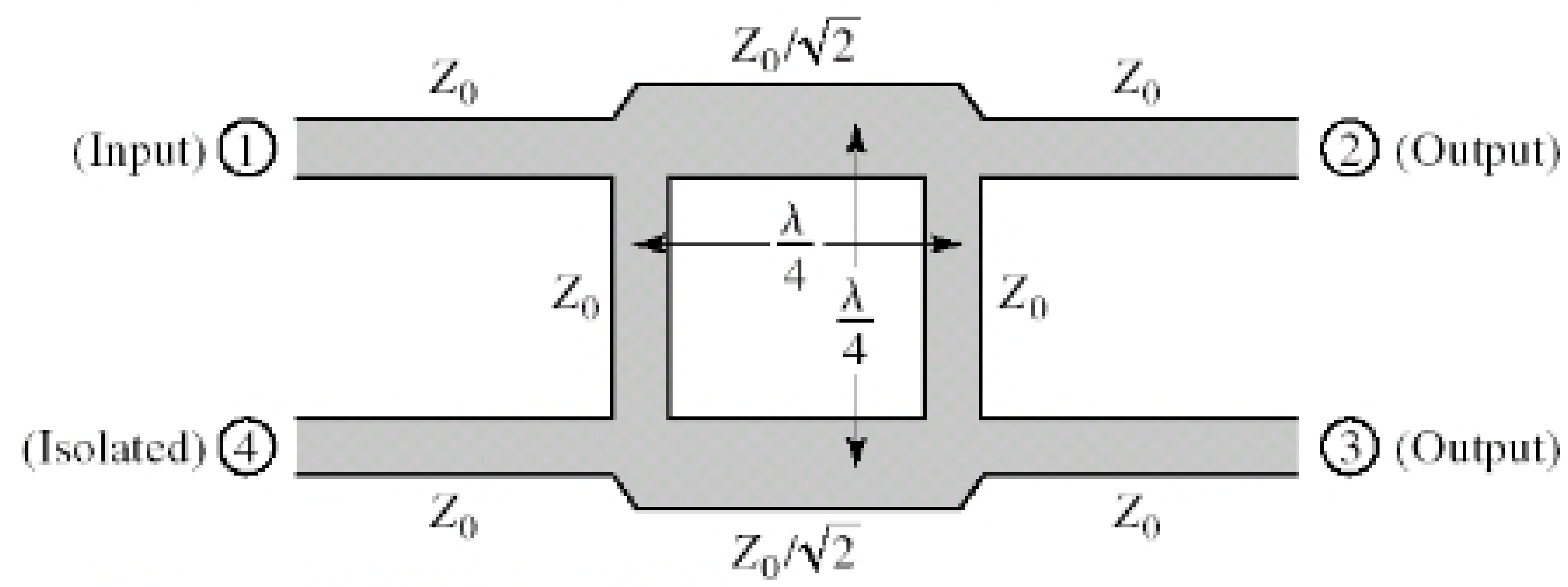


Figure 7.21 (p. 333) Geometry of a branch-line coupler.

Q: *Wow! How can we **analyze** such a complex circuit?*

A: Note that this circuit is **symmetric**—we can use **odd/even mode** analysis!

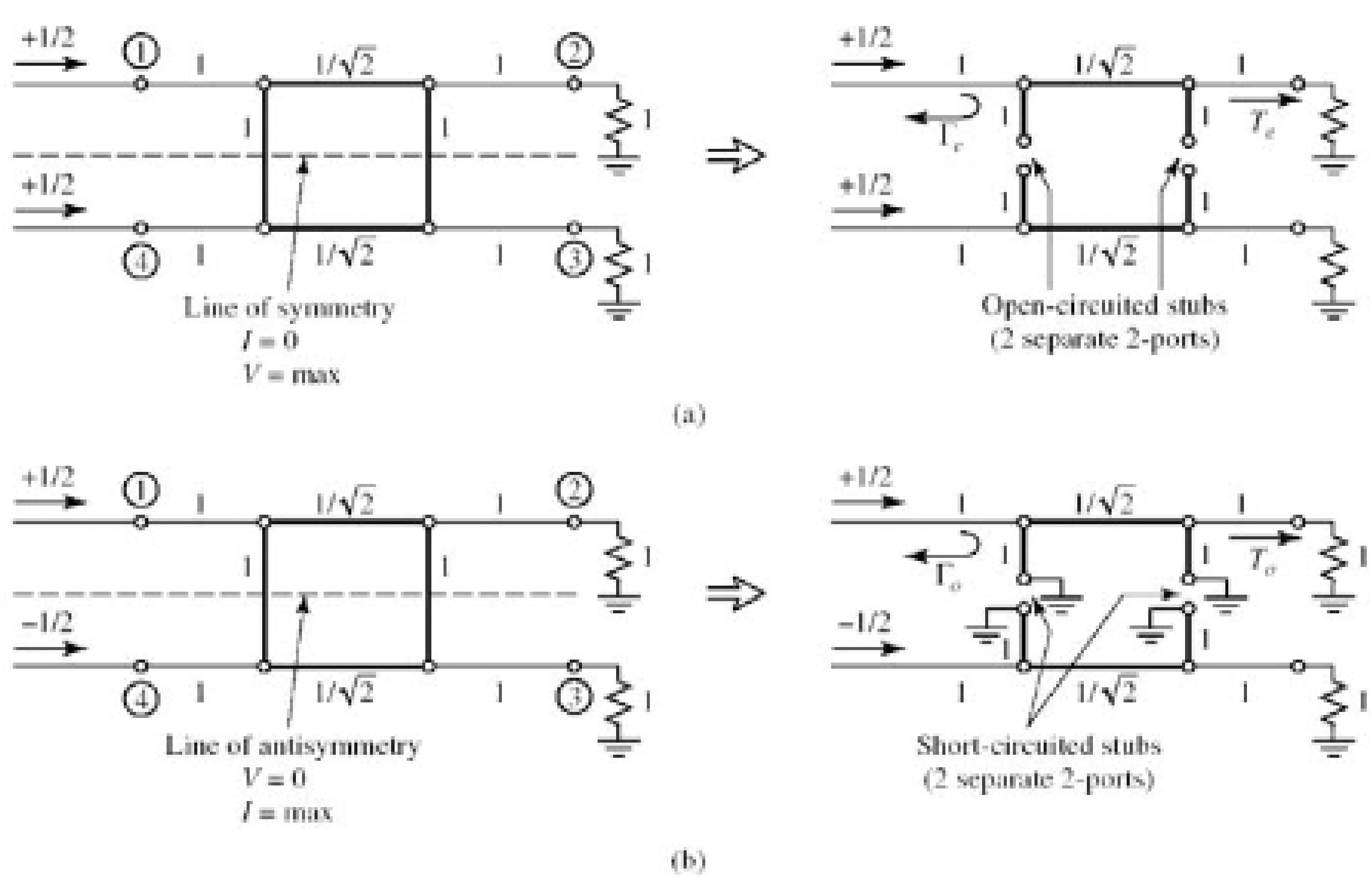


Figure 7.23 (p. 334) Decomposition of the branch-line coupler into even- and odd-mode excitations. (*a*) Even mode (*e*). (*b*) Odd mode (*o*).