

Homework 16 – Improper Integrals

1) Which of the following integrals is improper? Why or why not?

Do not evaluate the integrals.

a) $\int_0^2 \frac{dx}{x^{1/3}}$

b) $\int_1^{\infty} \frac{dx}{x^{0.2}}$

c) $\int_{-1}^{\infty} e^{-x} dx$

d) $\int_0^1 e^{-x} dx$

e) $\int_0^{\pi/2} \sec x dx$

f) $\int_0^{\infty} \sin x dx$

g) $\int_0^1 \sin x dx$

h) $\int_0^1 \frac{dx}{\sqrt{3-x^2}}$

i) $\int_1^{\infty} \ln x dx$

j) $\int_0^3 \ln x dx$

2) Determine whether each of the following improper integrals converges and, if so, evaluate it.

a) $\int_1^{\infty} \frac{dx}{x^{19/20}}$

b) $\int_{-\infty}^4 e^{0.0001t} dt$

c) $\int_0^5 \frac{dx}{x^{20/19}}$

d) $\int_0^4 \frac{dx}{\sqrt{4-x}}$

e) $\int_0^{\infty} \sin x dx$

f) $\int_0^1 \ln x dx$

g) $\int_0^1 \frac{\ln x}{x^2} dx$

h) $\int_{-\infty}^{\infty} xe^{-x^2} dx$

3) Use the comparison Test to determine whether or not each of the following integrals converges.

a) $\int_1^{\infty} \frac{1}{\sqrt{x^5+2}} dx$

b) $\int_0^1 \frac{e^x}{x^2} dx$

c) $\int_1^{\infty} \frac{1}{x^4 + \sqrt{x}} dx$

4) The solid region G obtained by rotating the region below the graph of $y = x^{-1}$ about the x-axis for $1 \leq x \leq \infty$ is called Gabriel's Horn. Show that the volume of G is finite and that the surface area of G is infinite.

5) The Laplace Transform of a function, $f(x)$, is the function $Lf(s)$ of the variable s defined by the improper integral

$$Lf(s) = \int_0^{\infty} f(x)e^{-sx} dx$$

if it converges.

a) Show that if $f(x) = C$, where C is a constant, then $Lf(s) = \frac{C}{s}$ for $s > 0$.

b) Compute $Lf(s)$, where $f(x) = e^x$ and $s > 1$.