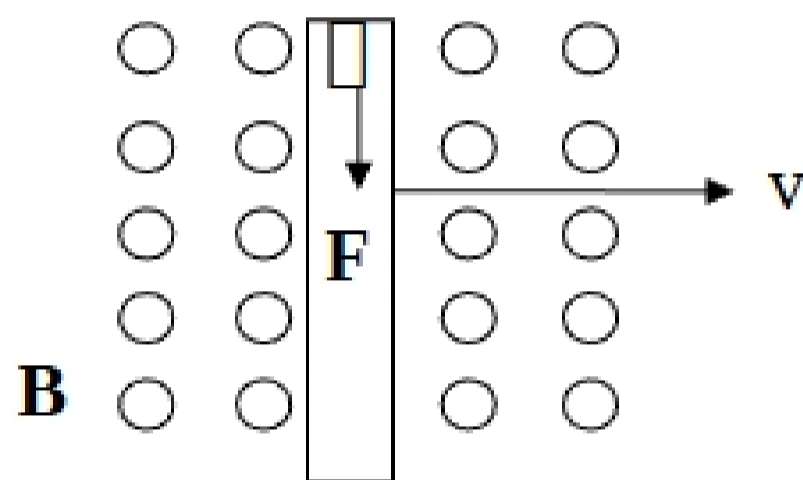


Electromagnetic Induction

Disclaimer: These lecture notes are not meant to replace the course textbook. The content may be incomplete. Some topics may be unclear. These notes are only meant to be a study aid and a supplement to your own notes. Please report any inaccuracies to the professor.

Induced EMF

Consider a conducting rod moving through a uniform magnetic field:



There is a Lorentz force acting on a charge q contained in the rod: $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$

The work to move the charge down the length of the rod (L) is given by:

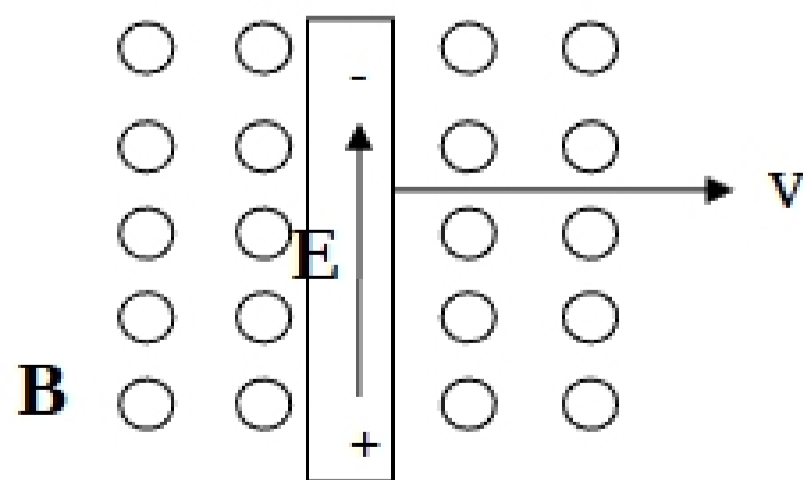
$$W = \int \mathbf{F} \cdot d\mathbf{s} = q \int \mathbf{v} \times \mathbf{B} \cdot d\mathbf{s} = qvBL$$

This work is supplied by the force that is pulling the rod, and raises the potential energy of the charge.

Thus the change in the electric potential difference in moving the charge down the length of the rod is the work done per unit charge:

$$dV = \frac{dU}{q} = \frac{W}{q} = vLB$$

Now asymptotically we cannot keep moving charge down the length of rod. The positive charge on one end (and negative on the other) will set up an electric field that will apply a force to each charge q in the opposite sense of the magnetic force.



After a very short time, the two forces will balance and the charges will be in equilibrium:

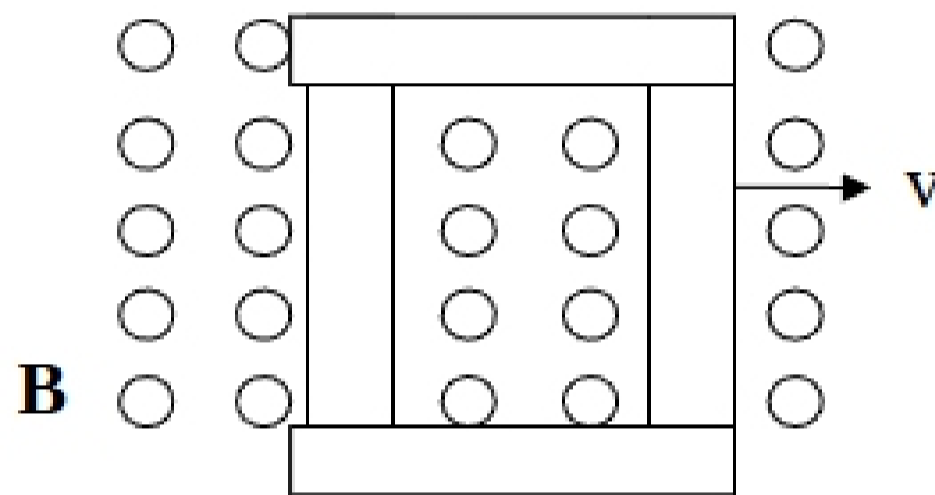
$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = 0 \Rightarrow \mathbf{E} = -\mathbf{v} \times \mathbf{B}$$

Thus we can calculate the electric potential difference another way from top to bottom:

$$dV = -\mathbf{E} \cdot d\mathbf{s} = \mathbf{v} \times \mathbf{B} \cdot d\mathbf{s} = vLB \quad (\text{same answer as before})$$

Thus we learn that moving a conductor in a magnetic field sets up an electric field within the conductor, that is we have induced an EMF.

Now consider a conducting loop in a magnetic field (not necessarily uniform from left to right):



There will be an induced EMF within both the left rod and the right rod from top to bottom:

$$e_L = vLB_L$$

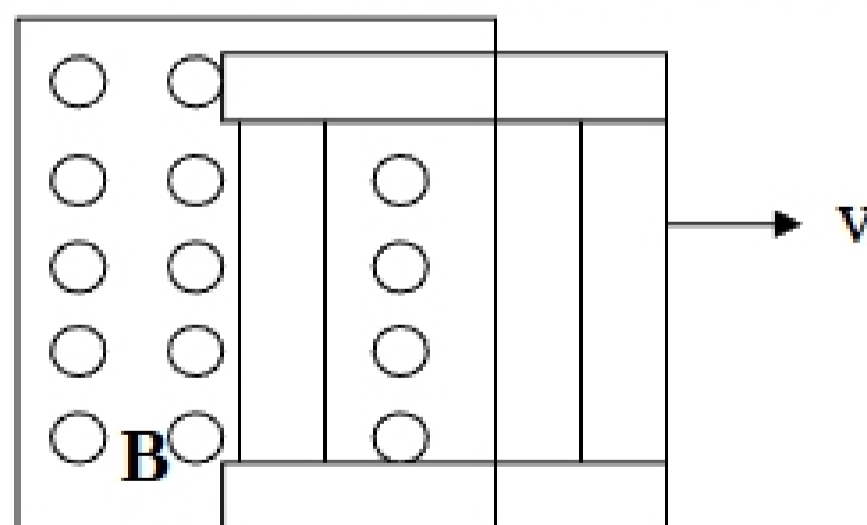
$$e_R = vLB_R$$

Where B_L and B_R is the magnetic field at the left and right bars, respectively. Now if consider the total EMF when traversing completely around the circuit we get:

$$e_{\text{tot}} = -\oint \mathbf{E} \cdot d\mathbf{s} = e_L - e_R = vL(B_L - B_R)$$

If the field at the left is not equal to that at the right, the induced EMF in the circuit is non-zero. Current will flow around the loop!

Consider the right side of the loop completely outside the magnetic field region:



Then $B_R=0$, and $e = vLB$.

If the loop has resistance R , the current $i = \frac{e}{R}$

And the power dissipated is $P = i^2 R = \frac{v^2 L^2 B^2}{R}$

This dissipated energy is supplied from the kinetic energy of the moving loop. If there is no additional external energy supplied, the kinetic energy reduces and the loop loses speed. This is referred to as magnetic braking or magnetic damping.

For example, a similar phenomenon occurs when a conducting sheet is pulled through a magnetic field. Eddy currents form in the sheet very much like current in a conducting loop. As the sheet has resistivity, power is dissipated, and the sheet will brake when passed through a magnetic field.

Now let's rewrite the form of the induced EMF for our example of a current loop with the right side outside the field and the left side within the magnetic field. The induced EMF can be written:

$$e = vLB = \frac{d}{dt} (W - vt) B$$

$$e = \frac{d}{dt} (B A)$$

Where L is the length of the vertical sides of the loop, W the total width, and $W-vt$ the width within the magnetic field region. If we define $L(W - vt)$ as the area A of the magnetic field region within the loop, then $B A$ is just the magnetic flux passing through the loop. This leads us to the following more general law.

Faraday's Law of Induction

Faraday's Law states that the change of the magnetic flux within a loop induces an EMF around the loop:

$e = - N \frac{dF_B}{dt}$ <p>N = number of turns of wire</p> $F_B = B A$ <p>is the flux through one loop of wire</p>

Thus if any of the following change with time:

1. The magnetic field strength
2. The area of the loop within the field
3. The orientation of the loop with respect to the field

There will be an induced EMF within the loop (an induced electric field).