

Inductive Proofs Must Have

- **Base Case (value):**
 - where you prove it is true about the base case
- **Inductive Hypothesis (value):**
 - where you state what will be assume in this proof
- **Inductive Step (value):**
 - show:
 - where you state what will be proven below
 - proof:
 - where you prove what is stated in the show portion
 - this proof must use the Inductive Hypothesis sometime during the proof

Prove this statement: $\sum_{i=1}^n i = \frac{n(n+1)}{2}$

Base Case (n=1): $\sum_{i=1}^1 i = 1$ $\frac{n(n+1)}{2} = \frac{1(1+1)}{2} = \frac{2}{2} = 1$

Inductive Hypothesis (n=p): $\sum_{i=1}^p i = \frac{p(p+1)}{2}$

Inductive Step (n=p+1):

Show: $\sum_{i=1}^{p+1} i = \frac{(p+1)((p+1)+1)}{2}$

Proof:(in class)

Variations

- $2+4+6+8+\dots+20 = ??$
- If you can use the fact:

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

- Rearrange it into a form that works.
- If you can't – you must prove it from scratch

Less Mathematical Example

- If all we had was 2 and 5 cent coins, we could make any value greater than 3.
- Base Case ($n = 4$):
- Inductive Hypothesis ($n=k$):
- Inductive Step ($n=k+1$):
show:
proof:

More Examples to be done in class

- $\forall n \in \mathbb{Z}^{\geq 1}, 3 \mid (n^3 - n)$

- $\forall n \in \mathbb{Z}^{\geq 0} \sum_{k=0}^n 2^k = 2^{n+1} - 1$

- Geometric Progression

$$\forall r \in \mathbb{R}^{>1} \forall a \in \mathbb{R} \forall n \in \mathbb{Z}^{\geq 0}, \sum_{j=0}^n ar^j = \frac{ar^{n+1} - a}{r-1}$$

Proving Inequalities with Induction

- Inductive Hypothesis
 - has the form $y < z$
- Inductive Step
 - needs to prove something of the form $x < z$
- Two methods for the proof part
 - use whichever you like
 - transitivity
 - find a value between (b)
 - prove that $b < z$
 - prove that $x < b$
 - book method
 - Substitute “unequals” as long as the signs don’t change
 - or
 - Add unequals to unequals as long as always adding correct sides