PRACTICE EXAMINATION ONE FOR FIRST MID-TERM

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1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total		110

- Start by writing your name in the above box and check your section in the box to the left.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or un-staple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

Problem 1) TF questions (20 points) No justifications needed

1) T F — A linear system with 2 equations and 3 unknowns has either infinitely many or no solutions.

2) T F — If S is an invertible matrix which contains the vectors $\vec{v}_1, ..., \vec{v}_n$ as columns, then $\vec{v}_1, ..., \vec{v}_n$ is a basis of $\mathbf{R}^n$.

3) T F — If A, B are given $n \times n$ matrices, then the formula $(A-B)(A+B) = A^2 - B^2$ holds.

4) T F — Suppose A is an $m \times n$ matrix, where $n < m$. If the rank of A is m, then there is a vector $y \in \mathbf{R}^m$ for which the system $Ax = y$ has no solutions.

5) T F — The matrix $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 3 & 3 & 3 \end{bmatrix}$ is invertible.

6) T F — The rank of an lower-triangular matrix equals the number of non-zero entries along the diagonal.

7) T F — The row reduced echelon form of a 3×3 matrix of rank 2 is one of the following $\begin{bmatrix} 1 & 0 & * \\ 0 & 1 & * \\ 0 & 0 & 0 \end{bmatrix}$ or $\begin{bmatrix} 1 & * & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

8) T F — The matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is a shear.

9) T F — For any matrix A, one has $\dim(\ker(A)) = \dim(\ker(\text{rref}(A)))$.

10) T F — If $\ker(A)$ is included in $\text{im}(A)$, then A is not invertible.

11) T F — There exists an invertible 3×3 matrix, for which 7 of the 9 entries are π.

12) T F — The set of functions $X = \{ f(x) = a \sin(x) + b \cos(x) + cx^2 + d \mid a, b, c, d \in \mathbb{R} \}$ is a linear subspace of all continuous functions on the real line.

13) T F — If A and B are $n \times n$ matrices, then AB is invertible if and only if both A and B are invertible.

14) T F — There exist matrices A, B such that A has rank 4 and B has rank 7 and AB has rank 5.

15) T F — There exist matrices A, B such that A has rank 2 and B has rank 7 and AB has rank 1.

16) T F — If for an invertible matrix A one has $A^2 = A$, then $A = I_n$.

17) T F — If an invertible matrix A satisfies $A^2 = I_2$, then $A = I_2$ or $A = -I_2$.

18) T F — The matrix $\begin{bmatrix} c-1 & -1 \\ 2 & c+1 \end{bmatrix}$ is invertible for every real number c.

19) T F — For 2×2 matrices A and B, if $AB = 0$, then either $A = 0$ or $B = 0$.

20) T F — If T is a rotation in space with an angle $\pi/6$ around the z axes, then the linear transformation $S(x) = T(x) - x$ is invertible.

Problem 2) (10 points)

Match each of matrices with one of the geometric descriptions below. You don't have to give explanations.

Matrix	Enter A-H here.
a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	
b) $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	
c) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$	
d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$	

Matrix	Enter A-H here.
e) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$	
f) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	
g) $\begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	
h) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$	

A) Shear along a plane.
B) Projection onto a plane.
C) Rotation around an axes.
D) Reflection at a point.
E) Projection onto a line.
F) Reflection at a plane.
G) Reflection at a line.
H) Identity transformation.

Problem 3) (10 points)

a) Write the matrix $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ as a product of a rotation and a dilation.

b) What is the length of the vector $\vec{v} = A^{100}e_1$, where e_1 is the first basis vector?