Volume integral

$$\int \vec{F}\, dV$$

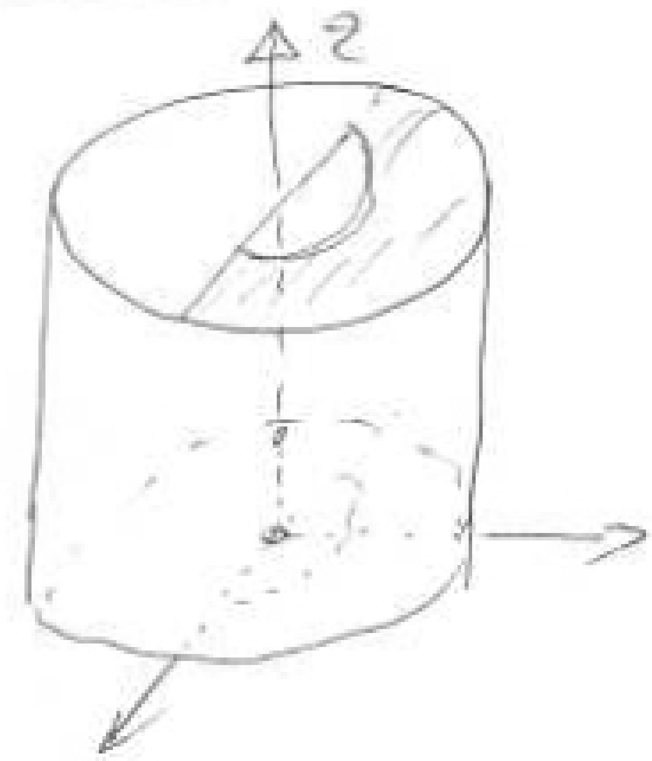
Cart : $dV = dx\,dy\,dz$

Cyl : $dV = \rho\, d\rho\, d\phi\, dz$

Sph : $dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$

Examples:

$\vec{F} = \rho \sin\frac{\phi}{2}\, a_x$



Vol: $\begin{cases} 1 \le \rho \le 2 \\ 0 \le \phi \le \pi \\ 0 \le z \le 1 \end{cases}$

$$\int \vec{F} dV = a_x \int_0^1 \int_0^\pi \int_1^2 \left(\rho \sin\frac{\phi}{2}\right) \rho\, d\rho\, d\phi\, dz$$

$$= a_x \left(\int_0^1 dz\right)\left(\int_0^\pi \sin\frac{\phi}{2}\right)\left(\int_1^2 \rho^2 d\rho\right)$$

$$= a_x \cdot 1 \cdot 2 \cdot \frac{7}{3}$$

$$= \frac{14}{3} a_x$$

What is the total charge within a spherical volume of radius 3 m? given $\rho_v = 2$ C/m^3

<u>Solution</u>

$$Q = \int \rho_v \, dV = \int_0^{2\pi} \int_0^{\pi} \int_0^{2} (2)\, r^2 \sin\theta \, dr\, d\theta\, d\phi$$

$$= \frac{64\pi}{3} \quad (C)$$

What is the charge within a spherical shell region of inner radius 1 m and outer radius 2 m?

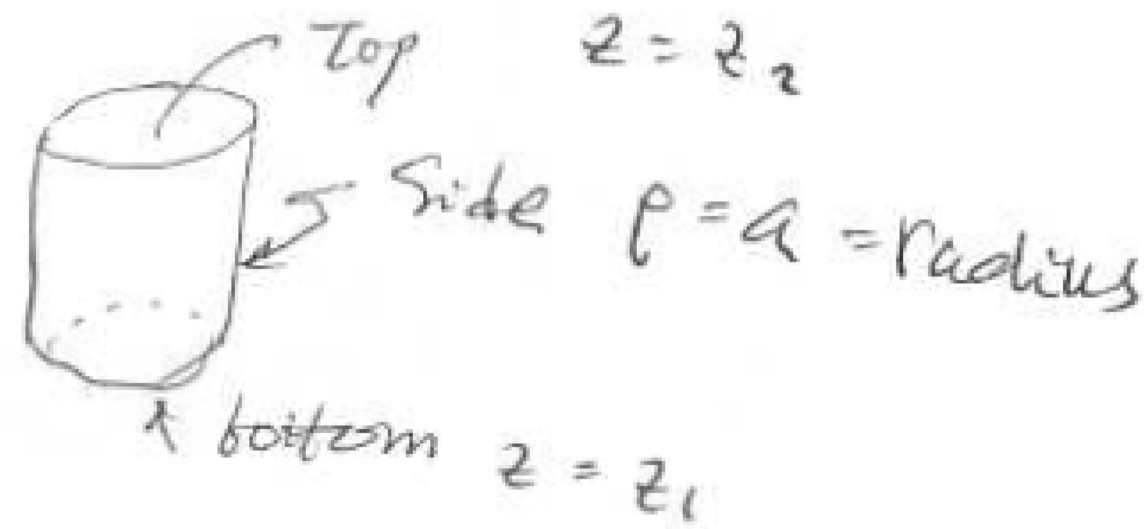
$$Q = \int \rho_v \, dv = \int_0^{2\pi} \int_0^{\pi} \int_1^{2} (2)\, r^2 \sin\theta \, dr\, d\theta\, d\phi$$

$$= \frac{4\pi}{3} \cdot 2 \cdot \left[2^3 - 1^3 \right] = \frac{56\pi}{3}$$

Surface integral $\quad \int \vec{F} \cdot d\vec{s}$

Cart: box faces

Cyl: top + bottom + side



$d\vec{s} = a_z \rho\, d\rho\, d\phi \qquad d\vec{s} = -a_z \rho\, d\rho\, d\phi \qquad d\vec{s} = a_\rho\, a\, d\phi\, dz$

Calculate the flux of a vector $\vec{F} = 3\rho\sin\phi\, a_\rho + 2\rho\cos\phi\, a_z$ through a cylindrical surface (closed) with

$a = 1\,m \quad (z=0) \rightarrow (z=1)$

$$\int \vec{F}\cdot d\vec{s} = \int_0^1 \int_0^{2\pi} (\vec{F})\cdot a_\rho \, a\, d\phi\, dz$$

$$+ \int_0^{2\pi}\int_0^1 \vec{F}\cdot a_z\, \rho\, d\rho\, d\phi + \int_0^{2\pi}\int_0^1 \vec{F}\cdot(-a_z \, d\rho\, d\phi)$$

$$= \left.\int_0^1\int_0^{2\pi} (3\rho\sin\phi)\,\rho\, d\phi\, dz\right]_{\rho=1}$$

$$+ \int_0^{2\pi}\int_0^1 (2\rho\cos\phi)\,\rho\, d\rho\, d\phi - \int_0^{2\pi}\int_0^1 (2\rho\cos\phi)\,\rho\, d\rho\, d\phi$$

$$= 0$$

Given $\vec{F} = a_r \dfrac{e^{-3r}}{r^2}$, evaluate $\oint_S \vec{F}\cdot ds$ on a spherical surface of radius 2

$$\int \vec{F}\cdot d\vec{s} = \left.\int_0^{2\pi}\int_0^{\pi} \left[\frac{e^{-3r}}{r^2} a_r\right]\cdot a_r\, r^2 \sin\theta\, d\theta\, d\phi\right]_{r=2}$$

$$= 2\pi\cdot 2\cdot e^{-6}$$

$$= 4\pi e^{-6}$$