Chapter 7 - Integration

Fundamental Theorem of Calculus

Part 1 - If F is an antiderivative of f (F' = f)

Then $\int_a^b f(x)dx = F(b) - F(a)$

Part 2 - If f is continuous

Then $\int_a^x f(t)dt$ is an antiderivative of f

Natural Log Rules

lne = 1

ln1 = 0

ln0 = undefined

ln(1/a) = -lna

ln(a/b) = lna - lnb

ln(ab) = lna + lnb

7.1 - *Integration by Substitution*

1). Let w be the 'inside' function
2). Reunite the integral as an integral in w
3). Evaluate the integral in w
4). convert w back to x

7.2 - Integration by Parts

$$\int udv = uv \int vdu$$

Choices for u

(in order of preference)

Logarithm
Inverse Trig
Polynomial
Exponential
Trig

7.3 - Tables of Integrals

Completing the Square

Trig Functions

$$sin^2x = \frac{1}{2}(1 - cos2x)$$

$$cos^2x = \frac{1}{2}(1 + cos2x)$$

sin(2x) = 2 sinx cosx
cos(2x) = $\cos^2$x - $\sin^2$x

7.4 - Algebraic Identities and Trigonometric Substitutions

Partial Fractions

Strategy for Integrating Rational Function: *P(x)/Q(x)*

1). If deg $P \geq$ deg Q, use algebraic long division and use partial fractions on the remainder

2). If Q(x) is a product of distinct linear factors, use partial fractions of the form: $\frac{A}{x-c}$

3). IF Q(x) has a repeated linear factor $(x-c)^n$, use partial fractions of the form:

$$\frac{A_1}{x+c} + \frac{A_2}{(x+c)^2} + \cdots + \frac{A_{n-1}}{(x-c)^n}$$

4). If Q(x) contains an unfactorable quadratic factor q(x), try a partial fraction of the form: $\frac{Ax+B}{q(x)}$

Trigonometric Substitutions

Sine Substitutions:

Use to simplify integrands involving $\sqrt{a^2 - b^2}$

Let $x = a\sin\theta$

Tangent Substitutions:

Use to simplify integrands involving a^2+x^2

Let $x = a\tan\theta$

7.5- Approximating Definite Integrals

Midpoint Rule

Trapezoidal Rule

$$\text{TRAP(n)} = \frac{\text{LEFT(n)} + \text{RIGHT(n)}}{2}$$

Under or Over Estimate

- If f is increasing on [a,b] then

$$\text{LEFT(n)} \leq \int_a^b f(x)dx \leq \text{RIGHT(n)}$$

- If f is decreasing on [a,b] then

$$\text{RIGHT(n)} \leq \int_a^b f(x)dx \leq \text{LEFT(n)}$$

Error in Left(n), Right(n)

$\sim 1/n$

Error in Mid(n), Trap(n)

$\sim 1/n^2$

Error in Simp(n)

$\sim 1/n^4$

- If f is ccd on [a,b] then

$$\mathrm{TRAP(n)} \leq \int_a^b \mathrm{f(x)dx} \leq \mathrm{MID(n)}$$

- If f is ccu on [a,b] then

$$\mathrm{MID(n)} \leq \int_a^b \mathrm{f(x)dx} \leq \mathrm{TRAP(n)}$$

Simpson's Rule

$$\frac{2\mathrm{MID(n)} + \mathrm{TRAP(n)}}{3}$$

7.6 - Improper Integrals

Occur when trying to integrate functions whose graphs are infinite in extent, either horizontally, vertically, or both.

Type 1 - Horizontally Infinite Region

Let f(x) be defined for $x \geq a$

If $\lim_{b\to\infty} \int_a^b f(x)dx$ exists and is finite, $\int_a^\infty f(x)dx$ converges

So $\int_a^\infty f(x)\,dx = \lim_{b\to\infty} \int_a^b f(x)dx$

Otherwise, we say $\int_a^\infty f(x)dx$ diverges.

Type 2 - Vertically Infinite Region

-vertical asymptotes

Suppose f(x) is defined on (a,b] and that $f(x) \to \infty$ or $f(x) \to -\infty$ as $x \to a$

If $\lim_{c\to a} \int_c^b \mathrm{f(x)dx}$ exists and is finite, then $\int_a^b f(x)dx$ converges

and $\int_a^b f(x)dx = \lim_{c\to a} \int_c^b f(x)dx$

Otherwise, $\int_a^b f(x)dx$ diverges.

Type 3 - Vertically and Horizontally

-infinite region

-basically a combination of types I and II which we handle by breaking up the interval of integration