CSE 167: Computer Graphics
Midterm
Winter, 2006

NAME: ________________________________

A. Vectors (4 points)

Given the following two vectors:

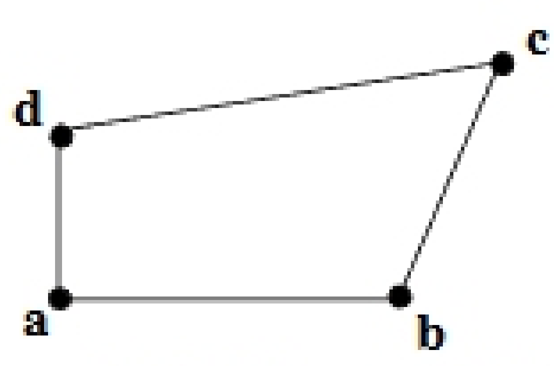
$$\vec{\mathbf{a}} = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix} \quad \vec{\mathbf{b}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Compute the following:

1. The magnitude of $\vec{\mathbf{a}}$ 5
2. The magnitude of $\vec{\mathbf{b}}$ 1
3. The dot product $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$ 4
4. The cosine of the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ 4/5 AKA .8

B. Normal (1 point)

You are looking down at a flat quadrilateral, defined by four points that all lie in a plane:



5. Write an expression for the normal to this quadrilateral. The normal should be pointing towards you (out of the paper).
   (b-a) X (d-a) / |(b-a) X (d-a)| and similar

C. Point and vector expressions (6 points)

For each of the following expressions involving points $\mathbf{a}$ and $\mathbf{b}$ and vectors $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$, circle "Point" if the result is a point, "Vector" if the result is a vector, or "Invalid" if the expression is not legal (AKA not *affine invariant*). Hint: consider what happens to the *w* component in homogeneous coordinates.

6.	$\mathbf{a}+\mathbf{b}$	Point	Vector	(Invalid)
7.	$\mathbf{a}-\mathbf{b}$	Point	(Vector)	Invalid
8.	$\vec{\mathbf{u}}-\vec{\mathbf{v}}$	Point	(Vector)	Invalid
9.	$\mathbf{a}+2\vec{\mathbf{u}}$	(Point)	Vector	Invalid
10.	$\frac{1}{3}\mathbf{a}+\frac{2}{3}\mathbf{b}$	(Point)	Vector	Invalid
11.	$\frac{1}{3}\mathbf{a}+\frac{2}{3}\vec{\mathbf{v}}$	Point	Vector	(Invalid)

D. Transformation matrix (2 points)

Consider a homogeneous affine transformation matrix $\mathbf{M}$, constructed from columns $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ and $\mathbf{d}$:

$$\begin{bmatrix} a_x & b_x & c_x & d_x \\ a_y & b_y & c_y & d_y \\ a_z & b_z & c_z & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

12. Write the result of transforming $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ by $\mathbf{M}$, in terms of $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ and $\mathbf{d}$. b+d

13. Suppose this matrix represents the local-to-world transform of an object that is not changing position, but is spinning about its local z-axis. Which column or columns ($\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ or $\mathbf{d}$) of the matrix will *not* change as the object spins?

c and d

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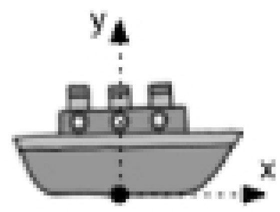
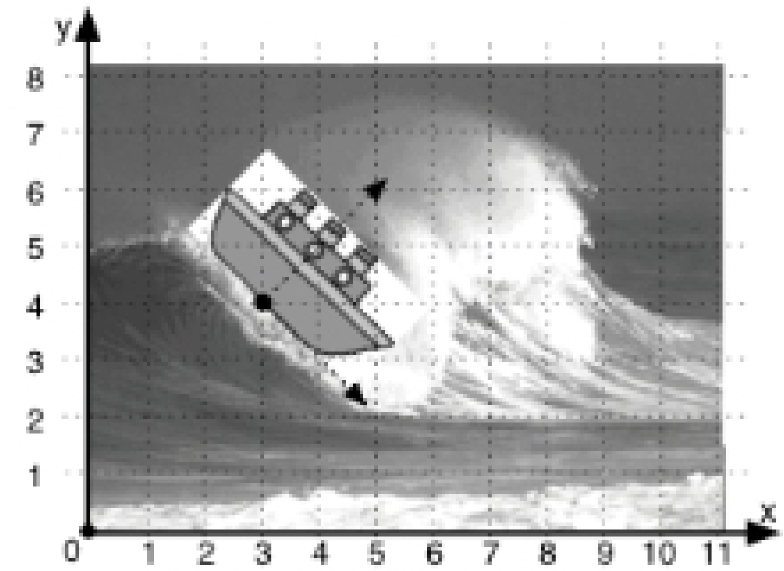
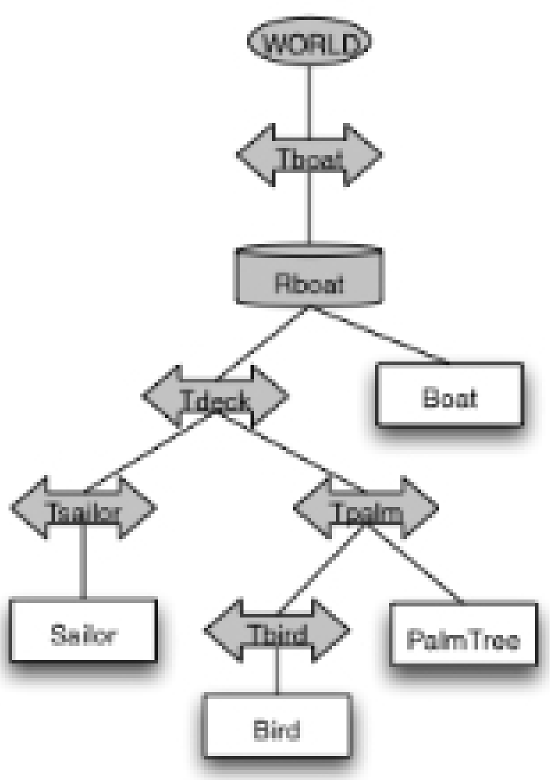
E. Scene Modeling (4 points)

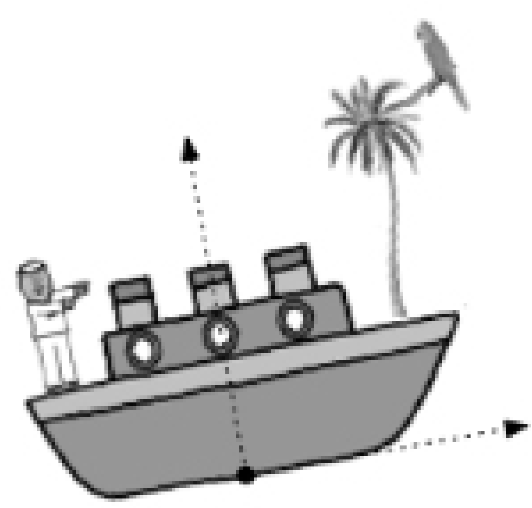




14. Write the local-to-world transform that places the boat (left) in world coordinates in the scene (right), expressed as a composition of 2D translation $\mathbf{T}(x, y)$ and rotation $\mathbf{R}(angle)$. Give numbers for the parameters of the transformations you use.

T(3,4) R(-45)





In this scene, a sailor and a palm tree are on the deck of the boat. The sailor's bird is on a branch of the tree. Write the following using composition of the local transformation matrices $\mathbf{T}_{boat}, \mathbf{R}_{boat}, \mathbf{T}_{deck}, \mathbf{T}_{sailor}, \mathbf{T}_{palm}$, and $\mathbf{T}_{bird}$ and their inverses as needed:

15. The object-to-world transform for the bird Tboat Rboat Tdeck Tpalm Tbird

16. The object-to-world transform for the sailor Tboat Rboat Tdeck Tsailor

17. The transform expressing the bird's frame in the sailor's coordinates

Tsailor^{-1} Tpalm Tbird