

Section 1.1 Introduction to functions

Definition of a function: A function, f , is a rule that assigns to each value, x , in a set D , a unique value, $f(x)$, in a set E .

The set of x values is called the domain, D , of f . The set of values of $f(x)$ for x in D is called the range of f .

Examples of ways to define a function are:

1] using a table. Let $D = \{1, 2, 3, 4, 5\}$

x	1	2	3	4	5
$f(x)$	2	4	6	8	10

or just listing the set of ordered pairs with x in the first coordinate and $f(x)$ in the 2nd;
 $\{(1, 2), (2, 4), (3, 6), (4, 8), (5, 10)\}$

2] with a rule. The above function is $f(x) = 2x$ for each x in $D = \{1, 2, 3, 4, 5\}$.

We know $f(3) = 6$ by reading the table or by substituting $x=3$ into the formula.

3] by describing in words.

Example: A right circular cylindrical can with top and bottom must have a volume of 60 cubic cm. S is the surface area of the can.

Find a rule for S as a function of the radius, r .

Call the height of the can h . Find S in terms of r and h . Then eliminate h using the given volume.

The lateral surface area of the cylinder is $2\pi r h$ and the areas of the top and bottom are both πr^2 . So $S = 2\pi r h + 2\pi r^2$.

The volume of the can is $\pi r^2 h = 60$ so $h = \frac{60}{\pi r^2}$.

Substituting into S we have $S(r) = 2\pi r \frac{60}{\pi r^2} + 2\pi r^2 = \frac{120}{r} + 2\pi r^2$.

The domain of the rule for $S(r)$ is all real numbers except 0. But the restricted domain that fits the context is all positive real numbers.

Finding domains. For now, you need these two rules:

- 1] We cannot take even roots of negative numbers.
- 2] We cannot divide by 0.

If a function is defined by a rule, give the domain as all x values where the rule is defined.

Ex. Find the domain of $f(x) = \sqrt{16 - x^2}$. Since we cannot take even roots of negative numbers, we must have $16 - x^2 \geq 0$. This is true only for the interval $[-4, 4]$. The square brackets mean that the endpoints are included.

Find the domain of each function.

$$1] \quad f(x) = \sqrt{2-x} \quad 2] \quad g(x) = \frac{1}{\sqrt{2-x}} \quad 3] \quad h(x) = \frac{1}{\sqrt{x^2-9}}$$

Answers:

$$1] \quad (-\infty, 2] \quad 2] \quad (-\infty, 2) \quad 3] \quad (-\infty, -3) \cup (3, \infty)$$

Piecewise functions: In piecewise functions, we follow different rules on different intervals.

$$\text{Ex. } f(x) = \begin{cases} x^2 - 9 & x \leq 0 \\ 2x + 4 & x > 0 \end{cases} \quad \text{Find } f(-2), \quad f(0), \quad f(1).$$

$-2 \leq 0$ so we use the first rule, $f(-2) = (-2)^2 - 9 = 4 - 9 = -5$

At 0, we also use the 1st rule since the 1st rule has 0 included. $f(0) = -9$

$1 > 0$ so at 1 we use the 2nd rule. $f(1) = 6$.

In graphing this function, we use a solid dot at the point (0, -9) to indicate that $f(0) = -9$. We use a small open circle at the point (0, 4) where the 2nd rule starts. The open circle indicates that (0, 4) is not included in the function graph.

Ex. The absolute value function $f(x) = |x|$ is the piecewise function,

$$f(x) = \begin{cases} -x & x < 0 \\ x & x \geq 0 \end{cases} \quad \text{For example, } f(-3) = 3 \text{ and } f(4) = 4.$$

Symmetry, Even and Odd functions:

If $f(-x) = f(x)$ for each x , then f is called an even function.

Any constant function and any even power of x is an even function. Any sum of even functions is also even.

Any polynomial with only even powers of x is even. For example $p(x) = 5 + 2x^2 - x^4$ is even.

If a function is even then its graph is symmetric about the y -axis.

If $f(-x) = -f(x)$ for each x , then f is called an odd function. Any polynomial with only odd powers of x is an odd function. For example $q(x) = x - 3x^3 + 4x^5$ is odd.

If a function is odd then its graph is symmetric about the origin.

The function $r(x) = 2 + x - x^2$ is neither even nor odd.

Increasing and Decreasing functions:

If the values of $f(x)$ increase as x increases, f is an increasing function. The graph rises as we move to the right. Any line with a positive slope is an increasing function.

If the values of $f(x)$ decrease as x increases, f is a decreasing function. The graph falls as we move to the right. Any line with a negative slope is a decreasing function.

Ex. $f(x) = x^2$ is decreasing on the interval $(-\infty, 0)$ and is increasing on the interval $(0, \infty)$.