Econ 311: Nonlinear Least Squares (NLLS) Estimator

A. Setup

1. Model of the population

Proceeding according to the analogy principle, suppose the true model of the population is

$$y_t = h(x_t, \beta) + u_t \qquad (1)$$

where x_t is a nonstochastic vector of exogenous variables. Unlike in the linear regression model, x, may not necessarily be of the same dimension as β. The $\{u_t\}$ are i.i.d. $\sim (0, \sigma^2)$. Since $h(x_t, \beta)$ is a nonlinear function of x_t and β, (1) is called the **nonlinear regression model.**

2. Choosing a criterion function

In this model, we expect that at the true value parameter value, $\beta = \beta_0$, the square of the error term, u_t is minimized. This would suggest that the criterion function be

$$S(\beta) = E_{\beta_0}[u_t]^2 = E_{\beta_0}[y_t - h(x_t, \beta)]^2, \quad (2)$$

which, at β_0, has the property P, where P is the minimum: $\beta_0 = \text{argmin}_\beta\, S(\beta)$.