10/16/14

Exam Friday! Ten Problems
Homework Due Thursday after Break.

Section 14.10: Partial Derivatives w/ constrained variables
-chain Rule & implicit differentiation.

w is dependent variable

$\left(\frac{\partial w}{\partial x}\right)_{yz}$ x y z are the independent variables

(x) $w = x^2 - y - z^2$
$x^2 + y^2 + z^2 = 9$

Find $\left(\frac{\partial w}{\partial z}\right)_x$

$\frac{\partial w}{\partial z} = 0 - \left(\frac{\partial y}{\partial z}\right) - 2z$

z partial of $w = x^2 - y - z^2$

z partial of $x^2 + y^2 + z^2 = 9$

$0 + 2y\frac{\partial y}{\partial z} + 2z = 0$

$-2y\frac{\partial y}{\partial z} = -2z$

$\frac{\partial y}{\partial z} = \left(\frac{-z}{y}\right)$

$\frac{\partial w}{\partial z} = -\left(\frac{-z}{y}\right) - 2z \rightarrow \left(\frac{\partial w}{\partial z}\right)_x = \frac{z}{y} - 2z$

evaluate at point $(x, y, z) = (-2, 2, 1)$

$\left(\frac{\partial w}{\partial z}\right)_x \Big|_{(-2,2,1)} = \frac{1}{2} - 2 = \boxed{-3/2}$

$w = x^3 + y^2 - z^3 + t^5$ and $xzt^2 = 8$

evaluate at point $(x, y, z, t) = (2, 2, 1, -2)$

Find $\left(\frac{\partial w}{\partial z}\right)_{xy}$

$\frac{\partial w}{\partial z} = 0 + 0 - 3z^2 + 5t^4 \frac{\partial t}{\partial z}$

$\frac{\partial}{\partial z}(xzt^2) = X\frac{\partial}{\partial z}(zt^2) = X\left(t^2 + z\frac{\partial}{\partial z}(t^2)\right)$

$\frac{\partial}{\partial z}(8) = 0 = X\left(t^2 + z\,2t\frac{\partial t}{\partial z}\right)$

$0 \neq X$

$0 = t^2 + 2zt\frac{\partial t}{\partial z} = 0$

$t\left(t + 2z\frac{\partial t}{\partial z}\right) = 0$

$t \neq 0$

$t + 2z\frac{\partial t}{\partial z} = 0$

$\frac{\partial t}{\partial z} = -t/2z$

$-3z^2 + 5t^4\left(-t/2z\right) = \left(\frac{\partial w}{\partial z}\right)_{xy} = \frac{-6z^3 - 5t^5}{2z}$

$\left(\frac{\partial w}{\partial z}\right)_{xy}\Big|_{\substack{x=2\\y=2\\z=1}} = -\left(\frac{6(1)^3 + 5(-2)^5}{2(1)}\right) = -\left(\frac{6 - 160}{2}\right) = \frac{152}{2} = 77$

Arrow Diagrams = worse than useless!!