Propositional Logic

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Formal Languages

Let S be an alphabet. Denote by S* the set of all strings over S, including the empty string. A formal language L over S is a subset of S*.

Our goal is to study the language Prop of propositional logic. This is a language over the alphabet $\Sigma = S \cup X$, where $S = \{ a, a_0, a_1, \ldots, b, b_0, b_1, \ldots \}$ and $X = \{\wedge, \vee, \oplus, \quad, \quad \}$.

We describe the language Prop using a grammar.

Grammar of Prop

$$\langle \text{formula} \rangle ::= \neg \langle \text{formula} \rangle$$

$$\mid (\langle \text{formula} \rangle \wedge \langle \text{formula} \rangle)$$

$$\mid (\langle \text{formula} \rangle \vee \langle \text{formula} \rangle)$$

$$\mid (\langle \text{formula} \rangle \oplus \langle \text{formula} \rangle)$$

$$\mid (\langle \text{formula} \rangle \quad \langle \text{formula} \rangle)$$

$$\mid (\langle \text{formula} \rangle \quad \langle \text{formula} \rangle)$$

$$\mid \langle \text{symbol} \rangle$$