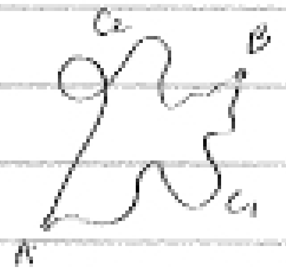
11/17/14

Vector field $\vec{F} = \langle M, N, P \rangle$

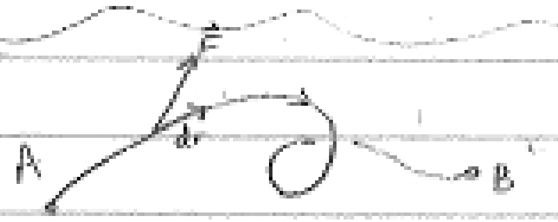
curve $C: \vec{r}(t) = \langle x(t), y(t), z(t) \rangle \qquad a \le t \le b$

$$\int_C \vec{F} \cdot d\vec{r} = \int_C \langle M, N, P \rangle \cdot \langle dx\ dy\ dz \rangle$$

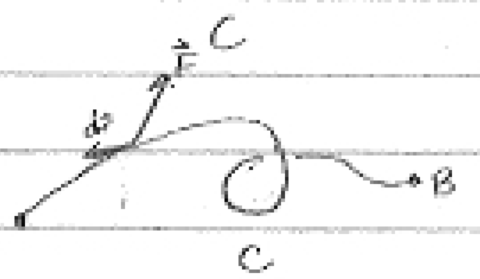
$$= \int_C \underbrace{M dx + N dy + P dz}_{1\ \text{form}}$$



If $C_1 = C_2$ they're conservative forces. Independent of path. Exact 1-forms.



$$\int_C \vec{F} \cdot d\vec{r}$$

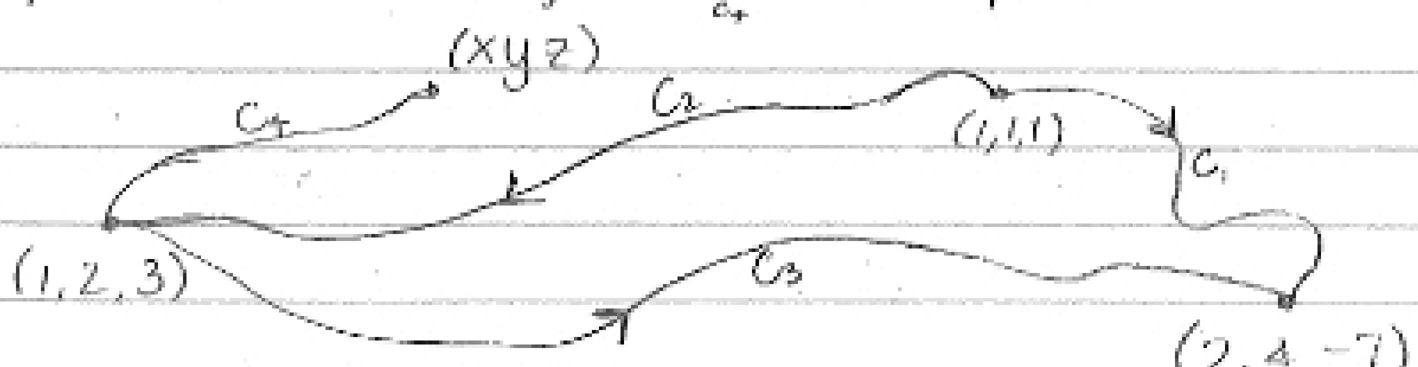


$$-\int_{-C} \vec{F} \cdot d\vec{r}$$

So $\int_C \vec{F} \cdot d\vec{r} = -\int_{-C} \vec{F} \cdot d\vec{r}$

Suppose $\int_C \vec{F} \cdot d\vec{r}$ is independent of the path & only depends on the endpoints.

(1,2,3) $\qquad f(xyz) = \int_{C_4} \vec{F} \cdot d\vec{r}$ potential function



$$\int_C \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r} + \int_{C_3} \vec{F} \cdot d\vec{r}$$

$$\int_C \vec{F} \cdot d\vec{r} = -\int_{C_4} \vec{F} \cdot d\vec{r} + \int_{C_5} \vec{F} \cdot d\vec{r}$$

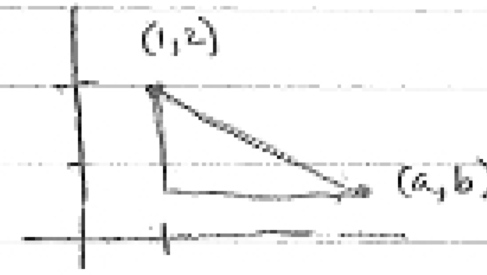
$f(2,4,-7) = f(2,4,-7) - f(1,1,1)$

EX) with no justification I claim

$(2x+4y)\,dx + (4x+8y)\,dy$ is exact (path indep., conservative)

Base Point $(1,2)$

Given any point in plane (a,b)



$$f(a,b) = \int_c (2x+4y)\,dx + (4x+8y)\,dy$$

$\vec{r}(0) = (1,2)$

$\vec{r}(1) = (a,b)$

$\langle 1,2\rangle + t\langle \quad \rangle = \langle x,y\rangle$ $\qquad x = 1+t(a-1)$

$\langle 1,2\rangle + \langle \quad \rangle = \langle a,b\rangle$ $\qquad y = 2+t(b-2)$

$\langle a-1, b-2\rangle$

$dx = a\,dt$

$dy = b\,dt$

$$f(a,b) = \int_0^1 [2(1+t(a-1)) + 4(2+t(b-2))a + 4(1+t(a-1)+8(2+t(b-2))b]\,dt$$

$$= \int_0^1 10a+20b+[2t(a-1)+4t(b-2)]a+[4t(a-1)+8t(b-2)]b\,dt$$

$$= (10a+20b)t + \frac{t^2}{2}(2(a-1)+4(b-2))a + (4(a-1)+8(b-2))b \Big|_{t=0}^{1}$$

$$= 10a+20b + \tfrac{1}{2}(2a^2-2a+4ab-8a+4ab-4b+8b^2-16b)$$

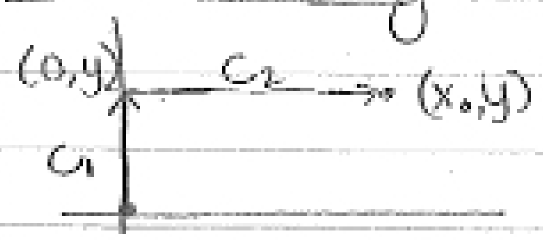
$$\boxed{= 5a+10b+a^2+4ab+4b^2}$$

Now if you plug in $\langle 1,2\rangle$ for (a,b) it should equal zero. (This one doesn't, messed up algebra)

The mistake was $dx = (a-1)\,dt$

$dy = (b-2)\,dt$

Easier Way : Same problem! Base pt $(0,0)$



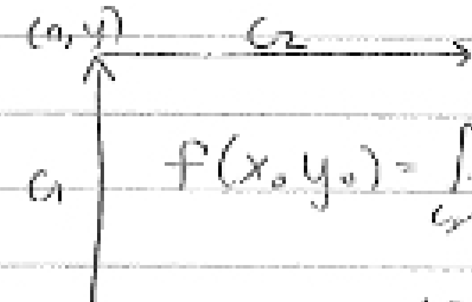
$$f(a,b) = \int (2x+4y)dx + (4x+8y)dy$$

$$f(x,y) = \int_{(0,0)}^{(x,y)} \vec{F} \cdot d\vec{r} = \int_{(0,0)\,(C_1)}^{(0,y)} F \cdot dr + \int_{(0,y)\,(C_2)}^{(x,y)} F \cdot dr$$

$x = t \qquad 0 \le t \le x_0$

$dx = dt$

$$\int_{C_2} F dr = \int_0^{x_0} F(t,y)\,dt$$



$$f(x_0,y_0) = \cancel{\int_{C_1} F dr} + \int_{C_2} F dr$$

constant

$$g(x) = \int_0^x F(t,y)\,dt$$

$$g'(x) = F$$

This is screwed up. Will fix tomorrow! Quiz