Lesson 26: Inverse Trigonometric Functions (Lesson 1 of 2)

(Cover 7.5, Start 8.4)	**Read:** Sections 7.5 & 8.4 **Do:** WebWork (Next Team Homework Due After Thanksgiving)

The most important points and skills for §7.5

- Students should be able to work with the notation of inverse trigonometric functions.
- Students should understand that inverse trigonometric functions give angles as output.
- Students should be able to use the inverse trig functions to find angles in right triangles.

The most important points and skills for §8.4

- Students should know the appropriate way to write inverse trigonometric functions.
- Students should know the domain and range for inverse sine, inverse cosine, and inverse tangent.
- Students should be able to calculate values using inverse trigonometric functions and use the graphs of cosine, sine, and tangent to help them determine if additional solutions are appropriate. For instance, students should be able to solve for x if $\sin(x) = 0.3$ using arcsin, and then use the graph of $\sin(x)$ to derive additional solutions over a given domain.
- Given a sinusoidal equation of the form $y = A\sin(B(x-h))+k$ (or $y = A\cos(B(x-h))+k$), students are able to solve for all x–values on a given domain that give a particular y–value. For example, students are able to draw an accurate sketch of the sinusoidal function over the given interval and a horizontal line at height C. Students should be able to use algebra, inverse trig functions, and graphical properties to calculate the x–coordinates of any intersection points in the domain given and state these as the solutions of $C = A\sin(B(x-h))+k$ (or $C = A\cos(B(x-h))+k$).

Note 1: Do not spend too long on Section 7.5. This section is an introduction to inverse trig functions, but we do not want to linger too long on right triangles. Although this is a comfortable place for students to start, the goal is for them to understand inverse trig functions beyond this context.

Note 2: Usually, the most difficult issue for students in this section is finding the additional solutions to a trigonometric equation using a graph and the symmetries that are present in the graph of a sinusoidal trigonometric function. Accurate and well-labeled drawings on your part, when you explain at the board, can go a long way towards helping to sort out what is going on in the students' minds.

Suggested Lesson Plan:

0-25 Return the second exam. Make sure to first put the scale, course median, and course mean on the board. Tell the class that the exams are uniformly graded so that all the grades were determined consistently throughout the course. Pass back the exams and tell the students

NOT to write on their exams. Inform students that changes are seldom made, but if the students do think a problem was misgraded, they should write on a **separate** sheet of paper what they think is wrong in the grading. Students are free to take their exams home that day, but once an exam leaves the classroom, NO changes will be made. Students who have potential grading problems should leave their exams with you along with their written sheet of paper. Do not spend time going through the answers of the exam. Solutions have been posted for students online. If you found your class consistently did poorly on 1-2 problems, you may discuss these, but DO NOT go through the entire exam. Students with more questions can see you in office hours.
Get back any exams with requested regrades *before* moving on to the lesson.

Segue as smoothly as you can into today's lesson. Outline the plan for the day.

25–35 Begin with an example like **Section 7.5 #16 on page 300**. Ask students what they know about the angle A. (Hopefully they will tell you the value of $\sin(A)$.) Then ask if this is enough to determine the exact measure of this angle. Use this as a way to define the inverse sine function on the domain $[0, 1]$. Then finish the problem. Be sure to give both exact and approximate answers. Remind students to be aware of whether their calculators are in degree or radian mode.

Define arccos and arctan for positive inputs similarly. For each of these functions, introduce both notations (e.g. $\sin^{-1}$ and arcsin). As with the notation for inverse functions in general, make sure to point out that, for example $\sin^{-1} x$ does NOT mean $1/\sin(x)$. To help clear up any confusion about this, ask the students to do **Section 7.5 #25 on page 300**.

35–45 Next move on to using inverse trig to solve equations. As examples, you could do **Section 7.5 #31 and 34 on page 300**. (As a preview of Section 8.4, ask students for other angles that satisfy the same equation.) Then have students work on **Section 7.5 #32 and 35 on page 300** in their groups. (#35 may pose more difficulty than you expect. Be prepared to help students figure out that they should combine the terms involving sin.)

45–55 Now move on to Section 8.4 and finding solutions more generally. You could begin by showing the students how to use their graphing calculators to find the solutions of a straightforward trigonometric equation such as $\sin(x) = -0.3$, $0 \leq x \leq 2\pi$. Students should plot both $y = \sin(x)$ and $y = -0.3$ and solve for intersection points. Make sure they find all of the solutions that are in the domain. Then ask the students to extend the domain of interest to $-3\pi \leq x \leq 3\pi$ and find all the solutions.

55–70 Segue into a mini-lecture about the general inverse sine and cosine functions by pointing out that we can use these functions to analytically solve equations such as $\sin(x) = -0.3$. Point out that these are the same functions as you were just using in right triangles but that you are now extending them to a larger domain.

Remind the class how the domain and range are related for inverse functions. Ask what an appropriate domain is for the inverse sine function. Explain why the range is restricted and make sure to define it. Ask them to use their calculators to solve for x if $\sin(x) = -0.3$. Explain that the calculator will only return the value that is contained in the range for the inverse function. Show them how they could find others by using the symmetry of sinusoidal graphs and periodicity.

Ask the students to do a short exercise to determine their understanding. Namely, ask them to solve for x given that $\cos(x) = -0.2$ on the domain $[0, 4\pi]$.

Upon completion of this example, have an interactive discussion about the appropriate domain and range for the inverse cosine function, making sure to define each.

70–80 Now move on to equations that require a bit of algebra, such as **Section 8.4 #25 and 26 on page 348**. Make sure that you show and explain each step carefully.

Summarize the lesson.

If you have extra time... you can discuss **Section 8.4 #52 on page 349**.