Lesson 26: Inverse Trigonometric Functions (Lesson 2 of 2)

(Finish 6.7)	**Read:** Section 8.1
Announce: EXAM 2 is Tuesday, March 22 at 6 pm	**Do:** WebWork, Team Homework

The most important points and skills for §6.7

- Students should know the appropriate way to write inverse trigonometric functions.
- Students should know the domain and range for inverse sine, inverse cosine, and inverse tangent.
- Students should be able to calculate values using inverse trigonometric functions and use the graphs of cosine, sine, and tangent to help them determine if additional solutions are appropriate. For instance, students should be able to solve for x if $\sin(x) = 0.3$ using arcsin, and then use the graph of $\sin(x)$ to derive additional solutions over a given domain.
- Given a sinusoidal equation of the form $y = A\sin(B(x-h)) + k$ (or $y = A\cos(B(x-h)) + k$), students are able to solve for all x–values on a given domain that give a particular y–value. For example, students are able to draw an accurate sketch of the sinusoidal function over the given interval and a horizontal line at height C. Students should be able to use algebra, inverse trig functions, and graphical properties to calculate the x–coordinates of any intersection points in the domain given and state these as the solutions of $C = A\sin(B(x-h)) + k$ (or $C = A\cos(B(x-h)) + k$).

Suggested Lesson Plan:

00–25 Use this time for a quiz or to do more examples from the last lesson. OR

If this is the day after the second exam in your class, return the exam. Make sure to first put the scale, course median, and course mean on the board. Tell the class that the exams are uniformly graded so that all the grades were determined consistently throughout the course. Pass back the exams and tell the students NOT to write on their exams. Inform students that changes are seldom made, but if the students do think a problem was misgraded, they should write on a **separate** sheet of paper what they think is wrong in the grading. Students are free to take their exams home that day, but once an exam leaves the classroom, NO changes will be made. Students who have potential grading problems should leave their exams with you along with their written sheet of paper. Do not spend time going through the answers of the exam. Solutions have been posted for students online. If you found your class consistently did poorly on 1-2 problems, you may discuss these, but DO NOT go through the entire exam. Students with more questions can see you in office hours.

Summarize the last lesson in 1-2 sentences. Outline today's lesson.

25–40 Move now into an example that requires more algebraic manipulation. Try an example like $4\sin\left(\frac{\pi}{2}(t-1)\right) - 3 = -2$. Ask the students to draw the graphs of $y = 4\sin\left(\frac{\pi}{2}(t-1)\right) - 3$ and $y = -2$ and mark all intersection points on the domain $[0, 8]$. (This is good review and practice with the material from Section 6.5.)

Now explain that we can use algebra and inverse trig functions to find *exact* solutions for t. Have the students work in their groups to find four solutions in the domain $[0, 8]$. When you go over the solution, make sure to demonstrate how to use algebra to solve for t and put ALL your steps on the board:

$$\begin{aligned} 4\sin\left(\frac{\pi}{2}(t-1)\right) - 3 &= -2 \\ 4\sin\left(\frac{\pi}{2}(t-1)\right) &= 1 \\ \sin\left(\frac{\pi}{2}(t-1)\right) &= \frac{1}{4} \end{aligned}$$

SO, $\frac{\pi}{2}(t-1)$ must be an angle whose sine is 1/4. ONE such angle is $\sin^{-1}(1/4)$. This gives

$$\begin{aligned} \frac{\pi}{2}(t-1) &= \sin^{-1}(1/4) \\ t-1 &= \frac{2}{\pi}\sin^{-1}(1/4) \\ t &= \frac{2}{\pi}\sin^{-1}(1/4) + 1 \end{aligned}$$

This gives ONE solution for t. To find the other solutions for t in the domain $[0, 8]$, there are several approaches, but please use the graph and symmetry.

Point out the difference between finding an exact solution and an approximation. Make sure to wrap up this example by showing how the answers correlate with the graph.

40–50 Now ask the students to do **Section 6.7 #57 on page 294**. Make sure that you discuss the solution fully and that every step is clearly shown on the board.

50–60 Move on to a real-world problem. Ask the students to do **Section 6.7 #51 on page 293**, but add part(d): "Find the t–value(s) for which $s = 50,000$. Interpret your answer." Be sure that the students are doing this algebraically and can find *exact* answers rather than just using the graph and their calculators.

60–70 Now on to the inverse tangent function. (This should feel very familiar.) Ask the to use their graphing calculators to find the solutions of $\tan(x) = 0.3$ over the domain $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Students should plot both $y = \tan(x)$ and $y = 0.3$ and solve for intersection points. Discuss the inverse tangent function, again noting that $\tan^{-1}$ does NOT mean $1/\tan(x)$. Then analytically solve the equation $\tan(x) = 0.3$, making sure to write the answer exactly and as an approximation. Remind the class how the domain and range are related for inverse functions and ask what the appropriate domain and range are for the inverse tangent function.

Now have the students extend their domain to $-\frac{3\pi}{2} < x < \frac{3\pi}{2}$ and ask them to find all solutions using their solution from before. Use the periodicity of $\tan(x)$.

70–80 End the class with a few short examples using arctangent. For example, have the students try to find all of the solutions to $4\tan(3x-1) = 10$ over the domain $[0, 2\pi]$. Make sure that the students are using correct algebra steps and are finding exact answers.

Summarize the lesson.

If you have extra time... other nice problems include **Section 6.7 #53 on page 294** and **#60 on page 298**.