TWO-SAMPLE T TEST
VARIANCE TEST
INTRO TO JMP

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Two-Sample t-Test

- Used to determine whether two population means are equal
- Variations
  - Paired samples vs. unpaired samples
    - In paired samples, there is a one-to-one correspondence between the values in the two samples. That is, for two random samples $X_1 = \{x_{11}, x_{12}, \ldots, x_{1n}\}$ and $X_2 = \{x_{21}, x_{22}, \ldots, x_{2n}\}$ x_{1i} corresponds to x_{2i} (i=1,2,…,n). In this case, the difference, $x_{1i} - x_{2i}$ is usually tested. If we can define $Y = X_1 - X_2$, then it becomes a one-sample t-test for Y in which $\mu_0 = 0$
    - In unpaired samples, there is no one-to-one correspondence between the values in the two samples
  - Equal variances vs. unequal variances of samples
    - The variances of the two samples may be assumed to be equal or unequal
    - Equal variances yield a simpler formula than unequal variances

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Two-Sample t-Test (Cont'd)

- Assumption
  - X_1 and X_2 are two random samples of independent observations, each from an underlying normal distribution $N(\mu_i, \sigma_i^2)$, where i =1, 2
- Null Hypothesis
  - H_0: $\mu_1 = \mu_2$
- Alternative Hypothesis
  - H_1: $\mu_1 \neq \mu_2$, or
  - H_1: $\mu_1 < \mu_2$, or
  - H_1: $\mu_1 > \mu_2$

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Two-Sample t-Test (Cont'd)

The formula for the unpaired two-sample t-test is

$$t = \frac{X_1 - X_2}{\sqrt{S_1^2/n_1 + S_2^2/n_2}} \quad (1) \qquad t \sim t(df)$$

← Standard error (SE)

If t is "near" zero, then it is consistent with the null hypothesis; otherwise, it is not.

$$df = \frac{(S_1^2/n_1 + S_2^2/n_2)^2}{(S_1^2/n_1)^2/(n_1-1) + (S_2^2/n_2)^2/(n_2-1)} \quad (2)$$

S_1, S_2 are the sample standard deviations of X_1 and X_2, respectively

n_1, n_2 are the sample sizes of X_1 and X_2, respectively

If equal variances are assumed, then (1) reduces to

$$t = \frac{X_1 - X_2}{S_p\sqrt{1/n_1 + 1/n_2}} \quad (3), \text{ where } S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}}$$

If $n_1 = n_2 = n$, then $S_p = \sqrt{\frac{S_1^2 + S_2^2}{2}}$

If equal variance are assumed, then (2) reduces to

$$df = n_1 + n_2 - 2 \quad (4)$$

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Search Interface Example I. You want to compare the effectiveness of two search interfaces – A vs. B. You have recruited 8 students for the experiment, each of whom perform 5 predetermined search tasks on each of the two interfaces. The following table shows the students' task performance scores in the experiment

		Student							
		S1	S2	S3	S4	S5	S6	S7	S8
Interface	A	5	4	2	3	2	1	5	4
	B	4	3	2	2	2	1	3	3

This is a Paired Two-Sample Two-Sided T Test

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Let X = (a student's performance score in interface A) − (his performance score in interface B).

Let μ_1 denote the mean of X

H_0: $\mu_1 = \mu_0 = 0$

H_1: $\mu_1 \neq \mu_0 = 0$

		Student							
		S1	S2	S3	S4	S5	S6	S7	S8
Interface	A	5	4	2	3	2	1	5	4
	B	4	3	2	2	2	1	3	3
	X	1	1	0	1	0	0	2	1

Set $\alpha = 0.05$, then the critical values are $t_{0.025}(7) = 2.365$ and $-t_{0.025}(7) = -2.365$ and the critical region is $t \geq 2.365$ or $t \leq -2.365$

$$\bar{X} = 0.75, S_{\bar{x}} = S_x / \sqrt{n} = 0.707 / \sqrt{8} = 0.25$$

$$t = \frac{\bar{X} - \mu_0}{S_{\bar{x}}} = \frac{0.75 - 0}{0.25} = 3 > 2.365$$

So we can reject the null hypothesis that interfaces A and B have the same effectiveness, i.e., there is evidence to support that interfaces A and B differ in their effectiveness

We can also estimate the p-value using the t-distribution table, $p \approx 0.02$

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