10/2/14

[illegible] MATH 1850

- local max/min in open region

  ① Find critical points
  ($\nabla f(P) = 0$ or undefined)

  ② Analyze critical points
  (2nd derivative test)

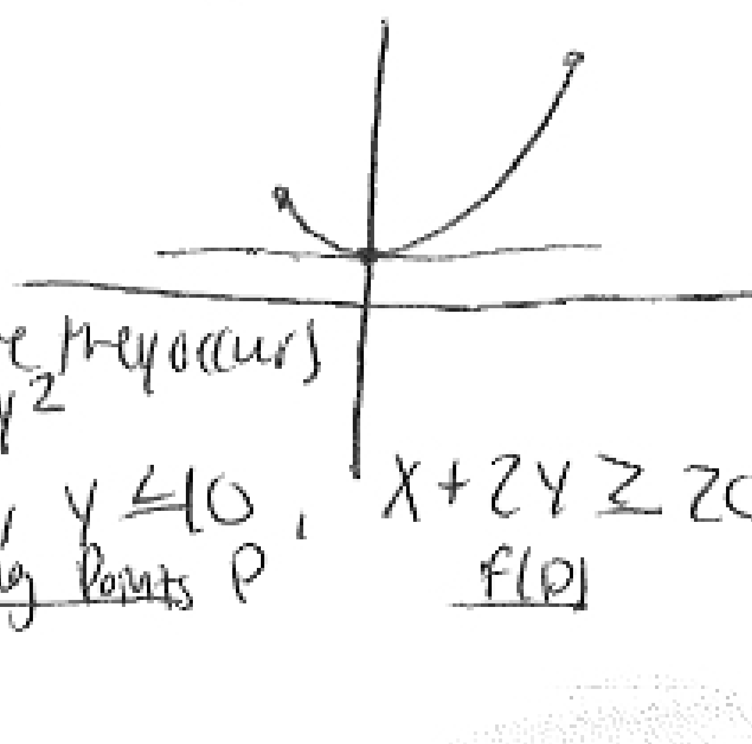
- global max/min on <u>compact</u> region
  - a region is compact if it is:
    ① finite in extent
    ② "closed"

<u>Then</u> if f is continuous on a compact region (Ω)
Then it has a global mass and a global min in Ω

① Finding <u>interesting</u> points
a) critical points ⓑ interesting boundary points

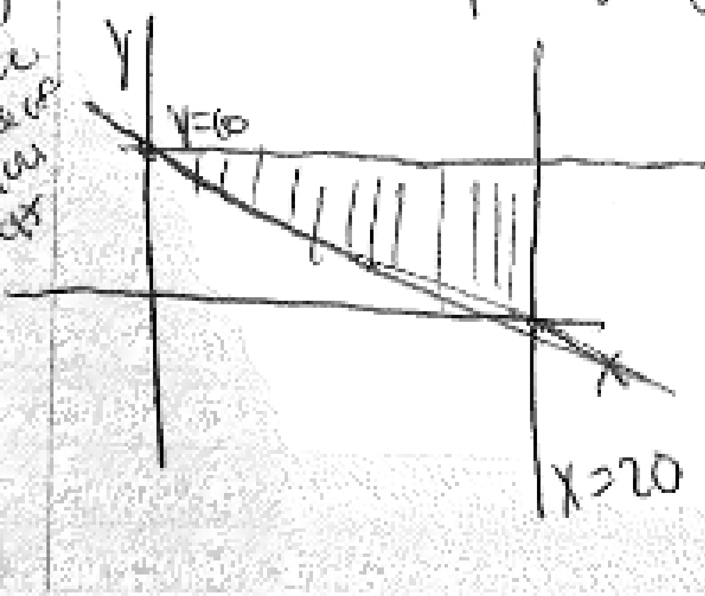
② Analyze interesting points
(Plug into "objective" function)

$f(x) = x^2 + 2 \quad \Omega = [-1, 5]$



<u>Ex:</u> Find global max & min (and where they occurs) of objective function $f(x,y) = x^2 + y^2$ on the (compact) region $x \le 20,\ y \le 10,\ x + 2y \ge 20$

Plug in ... to see ... side of ... angle you ... get



	<u>interesting Points P</u>	<u>f(P)</u>
interior		
edges	(20, 0)	400
	(0, 10)	100
	(4, 8)	80 ← min
corners	(20, 10)	500 ← max

Interior

Critical Point P

$\nabla f(P) = 0$ or undefined.

$\nabla f(x,y) = \langle 2x, 2y \rangle$

Set $\nabla f(x,y) = 0$

$2x = 0$ $\quad (x,y) = (0,0)$

$2y = 0$

Not in region !

Boundary 1

$x = 20 \quad 0 \le y \le 10$

$g(y) = f(20, y)$

$= 400 + y^2$

$g'(y) = 2y$

Set $2y = 0 \quad y = 0 \quad (20, 0)$

Boundary 2

$y = 10 \quad 0 \le x \le 20$

$h(x) = f(x, 10)$

$= x^2 + 100$

$h'(x) = 2x \quad 2x = 0 \quad x = 0 \quad (0, 10)$

Boundary 3

$x + 2y = 20$

$k(y) = f(20 - 2y, y)$

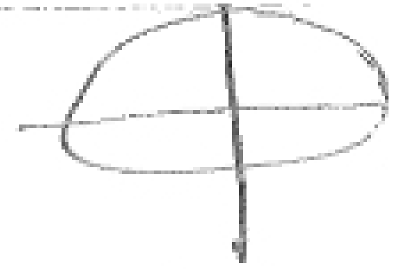
$= (20 - 2y)^2 + y^2$

$= 400 - 80y + 4y^2 + y^2 \quad = 400 - 80y + 5y^2$

Set $k'(y) = 0 \quad 0 = -80 + 10y \quad y = 8 \quad x = 4$

- f has a global max of 500 at (20,10) and global min ~~at~~ of 80 at (4,8)

~~Error~~



Ex: Find global max/min of objective function $f(x,y) = x^2 y$ subject to the constraint $x^2+2y^2=6$

$x = \sqrt{6-2y^2}$

$x^2 = 6-2y^2$

$g(y) = f(\underbrace{\quad}_{\text{whatever}}, y)$

$= (6-2y^2)\,y$

$= 6y - 2y^3$

$g'(y) = 6 - 6y^2$ Set $g'(y)=0$ $6-6y^2=0$

$y = \pm 1$

$y=1$ $x = \pm\sqrt{6-2} = \pm 2$

$y=-1$ $x = \cdots\cdots = \pm 2$

Interesting points P		f(P)
(2,1)	max	4
(-2,-1)		4
(2,-1)	min	-4
(-2,1)		-4

Ex: $f(x,y) = x^2+y^2$

$x^4 + 2y^4 = 1$ $x^2 = \sqrt{1-2y^4}$

$g(y) = f(\sqrt[4]{1-2y^4}, y)$

$= \sqrt{1-2y^4} + y^2$