CMSC250, Spring 2004 — Homework 4 Answers

Due Wednesday, February 25 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

1. Complete the following proofs using the method described in class (line numbers, rules, etc).

(a)

P1	$\forall x \in D\ P(x) \to (T(x) \vee Q(x))$
P2	$\forall y \in D\ Q(y) \vee (R(y) \wedge P(y))$
P3	$\forall z \in D\ (T(z) \wedge R(z)) \to S(z)$
$\therefore$	$\forall w \in D\ \sim Q(w) \to S(w)$

Answer:

Line	Statement	Rule	Lines Used
1	$Q(a) \vee (R(a) \wedge P(a))$	$\forall$ instantiation	P2
2	$\sim Q(a)$	Assume	—
3	$R(a) \wedge P(a)$	Disjunctive syllogism	1, 2
4	$P(a)$	Conjunctive simplification	3
5	$T(a) \vee Q(a)$	$\forall$ modus ponens	4, P1
6	$R(a)$	Conjunctive simplification	3
7	$T(a)$	Disjunctive syllogism	2, 5
8	$T(a) \wedge R(a)$	Conjunctive addition	6, 7
9	$S(a)$	$\forall$ modus ponens	P3, 8
10	$\sim Q(a) \to S(a)$	CCW w/out contra	2–9
11	$\forall w \in D\ \sim Q(w) \to S(w)$	$\forall$ generalization	10

Another way:

Line	Statement	Rule	Lines Used
1	$Q(a) \vee (R(a) \wedge P(a))$	$\forall$ instantiation	P2
2	$\sim (\sim Q(a) \to S(a))$	Assume	—
3	$\sim Q(a) \wedge \sim S(a)$	Definition of $\to$	2
4	$\sim Q(a)$	Conjunctive simplification	3
5	$\sim S(a)$	Conjunctive simplification	3
6	$\sim (T(a) \wedge R(a))$	$\forall$ modus tollens	5, P3
7	$\sim T(a) \vee \sim R(a)$	DeMorgan's law	6
8	$R(a) \wedge P(a)$	Disjunctive syllogism	1, 4
9	$R(a)$	Conjunctive simplification	8
10	$P(a)$	Conjunctive simplification	8
11	$\sim T(a)$	Disjunctive syllogism	7, 9
12	$\sim T(a) \wedge \sim Q(a)$	Conjunctive addition	4, 11
13	$\sim (T(a) \vee Q(a))$	DeMorgan's law	12
14	$\sim P(a)$	$\forall$ modus tollens	P1, 13
15	$\sim P(a) \wedge P(a)$	Conjunctive addition	14, 10
16	$\sim Q(a) \to S(a)$	CCW w/ contra	2–15
17	$\forall w \in D\ \sim Q(w) \to S(w)$	$\forall$ generalization	16

(b)

P1	$\forall t \in D\ (A(t) \to B(t)) \to (C(t) \vee D(t))$
P2	$\exists u \in D\ \sim A(u) \wedge (D(u) \to E(u))$
P3	$\forall v \in D\ \sim E(v) \to (C(v) \to A(v))$
$\therefore$	$\exists h \in D\ \sim B(h) \vee E(h)$

Answer:

Line	Statement	Rule	Lines Used
1	$\sim A(a) \wedge (D(a) \to E(a))$	$\exists$ instantiation	P2
2	$\sim A(a)$	Conjunctive simplification	1
3	$\sim A(a) \vee B(a)$	Disjunctive addition	2
4	$A(a) \to B(a)$	Definition of $\to$	3
5	$C(a) \vee D(a)$	$\forall$ modus ponens	P1, 4
6	$D(a) \to E(a)$	Conjunctive simplification	1
7	$\sim (\sim B(a) \vee E(a))$	Assume	—
8	$B(a) \wedge \sim E(a)$	Double negation, DeMorgan's law	7
9	$\sim E(a)$	Conjunctive simplification	8
10	$\sim D(a)$	Modus tollens	6, 9
11	$C(a)$	Disjunctive syllogism	5, 10
12	$C(a) \wedge \sim A(a)$	Conjunctive addition	2, 11
13	$\sim (\sim C(a) \vee A(a))$	Double negation, DeMorgan's law	12
14	$\sim (C(a) \to A(a))$	Definition of $\to$	13
15	$E(a)$	$\forall$ modus tollens, Double negation	P3, 14
16	$E(a) \wedge \sim E(a)$	Conjunctive addition	15, 9
17	$\sim B(a) \vee E(a)$	Closing cond world w/ contra	7–16
18	$\exists h \in D\ \sim B(h) \vee E(h)$	$\exists$ generalization	17

Another way:

Line	Statement	Rule	Lines Used
1	$\sim A(a) \wedge (D(a) \to E(a))$	$\exists$ instantiation	P2
2	$\sim A(a)$	Conjunctive simplification	1
3	$D(a) \to E(a)$	Conjunctive simplification	1
4	$\sim A(a) \vee B(a)$	Disjunctive addition	2
5	$A(a) \to B(a)$	Definition of $\to$	4
6	$C(a) \vee D(a)$	$\forall$ modus ponens	P1, 5
7	$C(a)$	Assume	—
8	$C(a) \wedge \sim A(a)$	Conjunctive addition	7, 2
9	$\sim (C(a) \to A(a))$	Definition of $\to$	8
10	$E(a)$	$\forall$ modus tollens	9, P3
11	$C(a) \to E(a)$	Closing cond world w/out contra	7–10
12	$E(a)$	Dilemma	6, 3, 11
13	$\sim B(a) \vee E(a)$	Disjunctive addition	12
14	$\exists h \in D\ \sim B(h) \vee E(h)$	$\exists$ generalization	13

(c)

P1	$\forall w \in D\ J(w) \to M(w)$
P2	$\forall x \in D\ M(x) \to ((N(x) \to \sim J(x)) \wedge \sim K(x))$
P3	$\forall y \in D\ N(y) \vee \sim L(y)$
P4	$\forall z \in D\ (J(z) \wedge K(z)) \vee (L(z) \wedge M(z))$
$\therefore$	$\forall q \in D\ \sim J(q)$

Answer:

Line	Statement	Rule	Lines Used
1	$N(a) \vee \sim L(a)$	$\forall$ instantiation	P3
2	$(J(a) \wedge K(a)) \vee (L(a) \wedge M(a))$	$\forall$ instantiation	P4
3	$J(a)$	Assume	—
4	$M(a)$	$\forall$ modus ponens	P1, 3
5	$(N(a) \to \sim J(a)) \wedge \sim K(a)$	$\forall$ modus ponens	P2, 4
6	$N(a) \to \sim J(a)$	Conjunctive simplification	5
7	$\sim N(a)$	Modus tollens, double neg	6, 3
8	$\sim L(a)$	Disjunctive syllogism	1, 7
9	$\sim K(a)$	Conjunctive simplification	5
10	$\sim J(a) \vee \sim K(a)$	Disjunctive addition	9
11	$\sim (J(a) \wedge K(a))$	double neg, DeMorgan's law	10
12	$L(a) \wedge M(a)$	Disjunctive syllogism	2, 11
13	$L(a)$	Conjunctive simplification	12
14	$L(a) \wedge \sim L(a)$	Conjunctive addition	8, 13
15	$\sim J(a)$	CCW w/ contra	3–14
16	$\forall q \in D \ \sim J(q)$	$\forall$ generalization	15

Another way:

Line	Statement	Rule	Lines
1	$M(a) \to ((N(a) \to \sim J(a)) \wedge \sim K(a))$	$\forall$ instantiation	P2
2	$\sim M(a) \vee ((N(a) \to \sim J(a)) \wedge \sim K(a))$	Definition of $\to$	1
3	$\sim M(a)$	Assume	—
4	$\sim J(a)$	$\forall$ modus tollens	P1, 3
5	$\sim M(a) \to \sim J(a)$	CCW w/out contra	3–4
6	$(N(a) \to \sim J(a)) \wedge \sim K(a)$	Assume	—
7	$N(a) \to \sim J(a)$	Conjunctive simp	6
8	$\sim K(a)$	Conjunctive simp	6
9	$\sim J(a) \vee \sim K(a)$	Disjunctive addition	8
10	$\sim (J(a) \wedge K(a))$	DeMorgan's law	9
11	$(J(a) \wedge K(a)) \vee (L(a) \wedge M(a))$	$\forall$ instantiation	P4
12	$L(a) \wedge M(a)$	Disjunctive syllogism	10, 11
13	$L(a)$	Conjunctive simp	12
14	$N(a) \vee \sim L(a)$	$\forall$ instantiation	P3
15	$N(a)$	Disjunctive syllogism	13, 14
16	$\sim J(a)$	Modus ponens	15, 7
17	$[(N(a) \to \sim J(a)) \wedge \sim K(a)] \to \sim J(a)$	CCW w/out contra	6–16
18	$\sim J(a)$	Dilemma	2, 5, 17
19	$\forall q \in D \ \sim J(q)$	$\forall$ generalization	18

2. Translate each of the following into formal language using the sets and predicates given.

   (a) Exactly two people completely understand quantum physics. ($U = \{\text{universal set}\}$, $P(m) = m$ is a person, $Q(n) = n$ completely understands quantum physics.)

   Answer:
   $\exists a, b \in U \ P(a) \wedge Q(a) \wedge P(b) \wedge Q(b) \wedge a \neq b \wedge \forall x \in U \ (P(x) \wedge Q(x)) \to (a = x \vee b = x)$