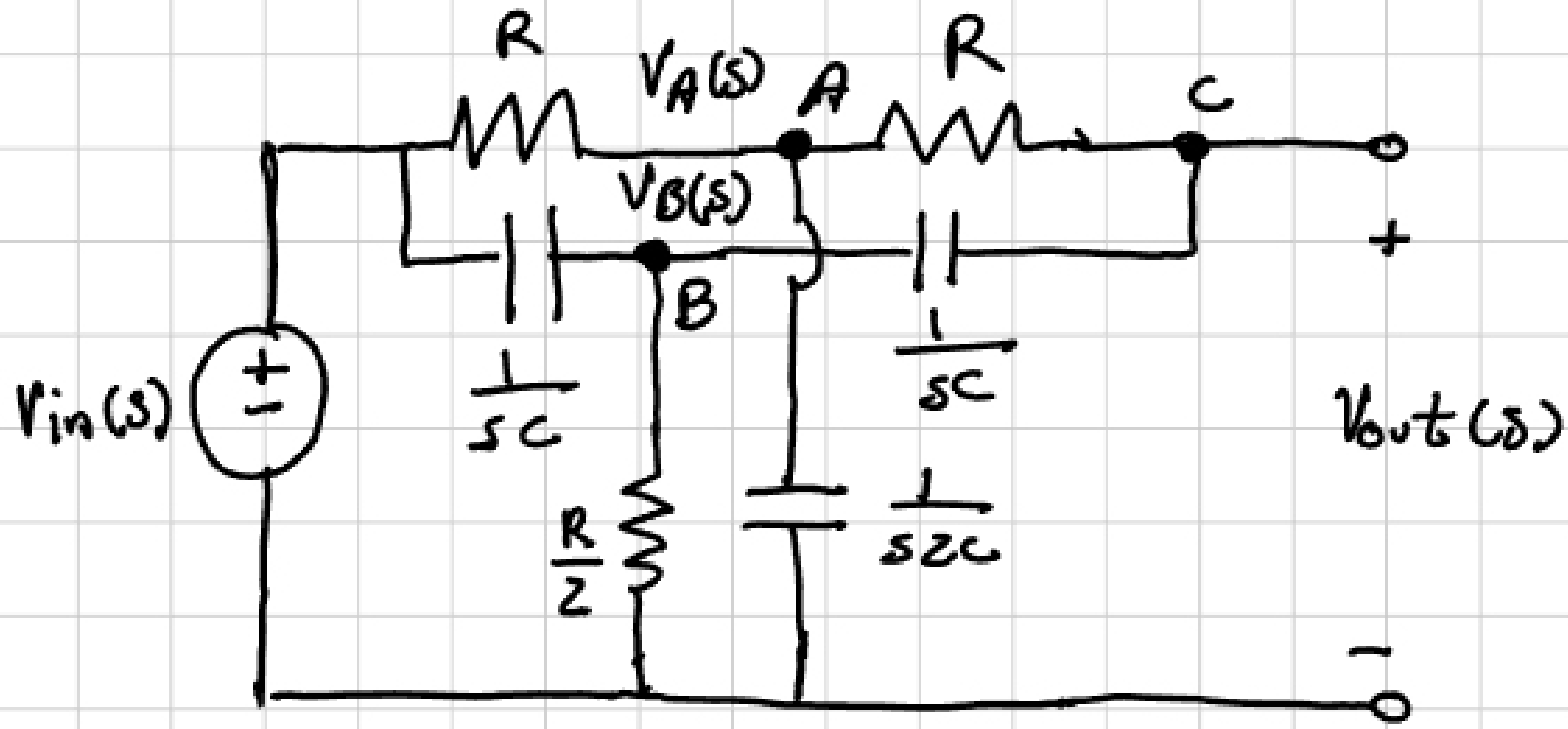
Note Title

## Problem 1

1.




Using KCL:

Node A: $$\frac{V_A - V_{in}}{R} + \frac{V_A}{\frac{1}{s2C}} + \frac{V_A - V_{out}}{R} = 0 \qquad (1)$$

Node B: $$\frac{V_B - V_{in}}{\frac{1}{sC}} + \frac{V_B}{\frac{R}{2}} + \frac{V_B - V_{out}}{\frac{1}{sC}} = 0 \qquad (2)$$

Node C: $$\frac{V_{out} - V_A}{R} + \frac{V_{out} - V_B}{\frac{1}{sC}} = 0 \qquad (3)$$

From Equation (1), determine $V_A$ in terms of $V_{out}$ and $V_{in}$:

$$V_A\left[\frac{2}{R} + s2C\right] = \frac{V_{out} + V_{in}}{R} \Rightarrow V_A = \frac{1}{2}\,\frac{V_{out} + V_{in}}{sRC + 1} \qquad (4)$$

## Problem 1

1. From Equation (2) determine $V_B$ in terms of $V_{out}$ and $V_{in}$:

$$V_B\left[s2C + \frac{2}{R}\right] = sC\left(V_{in} + V_{out}\right) \Rightarrow$$

$$V_B = \frac{1}{2}\frac{sRC}{sRC+1}\left(V_{out} + V_{in}\right) \qquad (5)$$

In Equation (3), eliminate $V_A$ and $V_B$ in favor of $V_{in}$ and $V_{out}$ using equations (4) and (5)

$$V_{out}\left[sRC+1\right] = V_A + sRC\,V_B \qquad \text{(from Eqn (3))}$$

$$V_{out}\left[sRC+1\right] = \frac{1}{2}\frac{V_{out}+V_{in}}{sRC+1} + \frac{1}{2}\frac{(sRC)^2}{sRC+1}\left(V_{out}+V_{in}\right)$$

$$V_{out}\left[sRC+1 - \frac{1/2}{sRC+1} - \frac{1/2\,(sRC)^2}{sRC+1}\right]$$

$$= V_{in}\left[\frac{1/2}{sRC+1} + \frac{1/2\,(sRC)^2}{sRC+1}\right]$$

$$V_{out}\left[2(sRC+1)^2 - 1 - (sRC)^2\right] = V_{in}\left[1 + (sRC)^2\right]$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{(sRC)^2 + 1}{(sRC)^2 + s4RC + 1}$$

$$\boxed{\frac{V_{out}(s)}{V_{in}(s)} = \frac{s^2 + \frac{1}{(RC)^2}}{s^2 + s\frac{4}{RC} + \frac{1}{(RC)^2}}}$$

Problem 1

2. In terms of $\tau = RC$, the network transfer function is

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{s^2 + \frac{1}{\tau^2}}{s^2 + s\frac{4}{\tau} + \frac{1}{\tau^2}}$$

The zeros of the transfer function satisfy

$$s^2 + \frac{1}{\tau^2} = 0$$

and are located at

$$s = +j/\tau \quad \text{and} \quad -j/\tau.$$

The poles of the transfer function satisfy

$$s^2 + s\frac{4}{\tau} + \frac{1}{\tau^2} = 0$$

and are located at

$$s = -\frac{2}{\tau} \pm \sqrt{\left(\frac{2}{\tau}\right)^2 - \frac{1}{\tau^2}}$$

$$= -\frac{2}{\tau} \pm \frac{\sqrt{3}}{\tau} = -\frac{2 \pm \sqrt{3}}{\tau}$$

Pole-zero map of $V_{out}/V_{in}(s)$: