Note Title

Problem 7

1. $G_p(s) = Y(s)/U(s) = \dfrac{2s-1}{s^4+6s^3+4s^2+2s+1}$

To obtain an ODE representation, cross multiply and determine the inverse Laplace transform.

$$s^4Y + 6s^3Y + 4s^2Y + 2sY + Y = 2sU - U$$

$$\boxed{\frac{d^4y}{dt^4} + 6\frac{d^3y}{dt^3} + 4\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 2\frac{du}{dt} - u}$$

2. $$\frac{Y}{U} = \underbrace{\left(\frac{1}{s^4+6s^3+4s^2+2s+1}\right)}_{P/U}\underbrace{(2s-1)}_{Y/P}$$

From P/U:

$$s^4P + 6s^3P + 4s^2P + 2sP + P = U$$

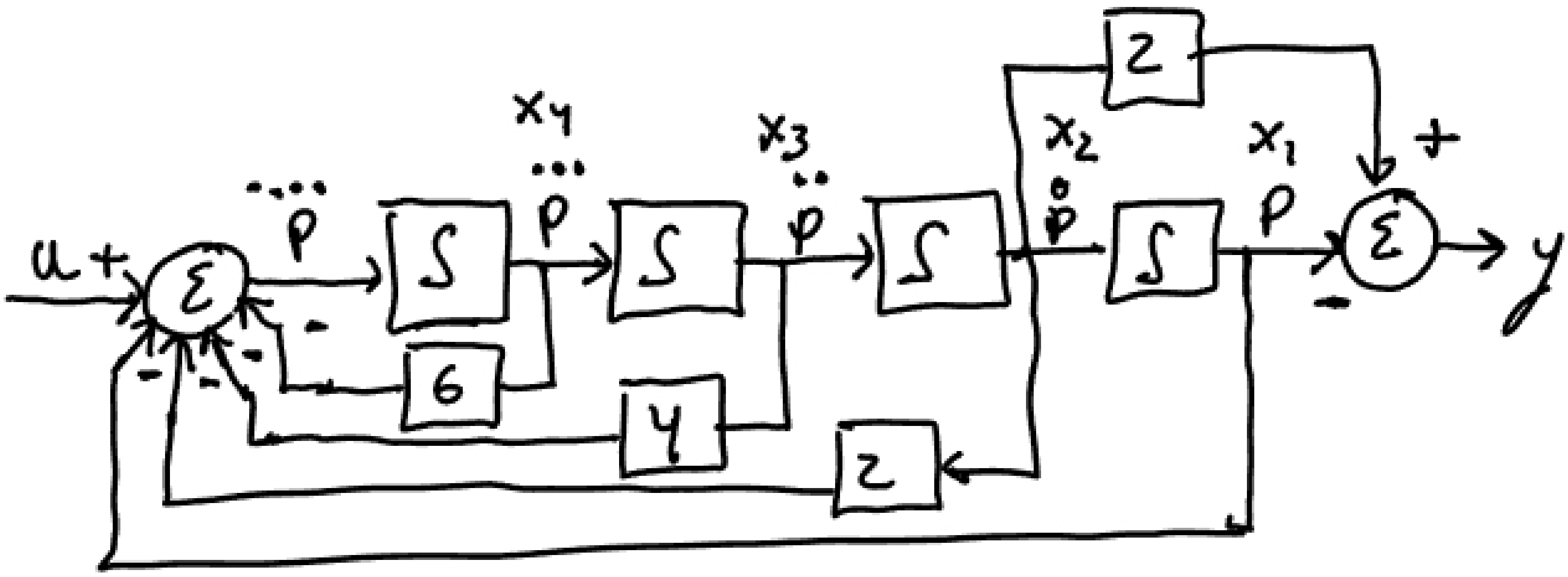
$$\Rightarrow \ddddot{p} = -6\dddot{p} - 4\ddot{p} - 2\dot{p} - p + u$$

From Y/P:

$$Y = 2sP - P \Rightarrow y = 2\dot{p} - p$$

Problem 7

2. Construct the all-integrator block diagram using a chain of four integrators with $\ddddot{\rho}$ as the input



3. Using the state-variable assignments
$x_1 = \rho$, $x_2 = \dot{x}_1$, $x_3 = \dot{x}_2$, $x_4 = \dot{x}_3$,

we obtain the controllable canonical state-space representation:

$$\boxed{\begin{aligned} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} &= \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -2 & -4 & -6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 6 \\ 0 \\ 0 \\ 1 \end{pmatrix} u \\ y &= (-1, 2, 0, 0) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + 0\, u \end{aligned}}$$

Problem 8

1. Given the system represented by the ODE $\ddot{y} + 5\dot{y} + 6y = 4u$, we need to find the zero-input and zero-state response for $y(0) = 1$, $\dot{y}(0) = -1$, and $u(t) = e^{-t} u_0(t)$.

Take the Laplace transform of both sides of the ODE:

$$s^2 Y(s) - s y(0) - \dot{y}(0) + 5(sY(s) - y(0)) + 6Y(s) = 4\,U(s)$$

$$Y(s)\left[s^2 + 5s + 6\right] = s y(0) + \dot{y}(0) + 5y(0) + 4\,U(s)$$

$$Y(s) = \underbrace{\frac{s y(0) + \dot{y}(0) + 5y(0)}{s^2 + 5s + 6}}_{Y_{zi}(s)} + \underbrace{\frac{4}{s^2 + 5s + 6}\,U(s)}_{Y_{zs}(s)}$$

First find the zero-input response using $y(0) = 1$ and $\dot{y}(0) = -1$.

$$Y_{zi}(s) = \frac{s+4}{s^2 + 5s + 6} = \frac{s+4}{(s+2)(s+3)}$$

$$= \frac{A}{s+2} + \frac{B}{s+3}$$

$$A = \cancel{s+2}\, \frac{s+4}{\cancel{(s+2)}(s+3)}\bigg|_{s=-2} = 2$$

$$B = \cancel{s+3}\, \frac{s+4}{(s+2)\cancel{(s+3)}}\bigg|_{s=-3} = -1$$