CMSC 250 Quiz #10 Answers Wed., April 7, 2004

Write all answers legibly in the space provided. The number of points possible for each question is indicated in square brackets – the total number of points on the quiz is 30, and you will have exactly 15 minutes to complete this quiz. You may not use calculators, textbooks or any other aids during this quiz.

1. [12 pnts.] Assuming Σ is the set $\{a, b, c\}$ do each of the following:

   a. Give the value of Σ^2.

   $$\Sigma^2 = \{aa, ab, ac, ba, bb, bc, ca, cb, cc\}$$

   b. Give the power set of Σ.

   $$P(\Sigma) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$$

   c. Assuming $A = \{1, 2\}$ - give $A \times \Sigma$.

   $$A \times \Sigma = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$$

2. [8 pnts.] Give the lists of elements in each of the sets (A and B) assuming $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$ and $A \cap B = \{3, 6, 9\}$.

   $$A = \{1, 5, 7, 8, 3, 6, 9\}$$

   $$B = \{2, 10, 3, 6, 9\}$$

3. [10 pnts.] Prove or give a counter example to the following. For all sets A, B and C. If $A \subseteq B$ and $B \cap C = \emptyset$, then $A \cap C = \emptyset$.

Suppose A, B and C are arbitrary sets.

-DIRECT METHOD

Assume $A \subseteq B$ and $B \cap C = \emptyset$
and an arbitrary x such that $x \in A$

Since $x \in A$ and $A \subseteq B$, by the definition of subset $x \in B$.
Since $B \cap C = \emptyset$, B and C are disjoint sets.
Since B and C are disjoint sets and $x \in B$, then $x \notin C$.

This means that $\forall x \in U, x \in A \rightarrow x \notin C$.
By the definition of complement, this is the same as: $\forall x \in U, x \in A \rightarrow x \in C^c$

By the definition of subset, this means: $A \subseteq C^c$

Therefore every member of A would need to be in C^c.
Since C and C^c are disjoint, A and C would also be disjoint.

Therefore, $A \cap C = \emptyset$. QED

- USING CONTRADICTION

Assume $x \in (A \cap C)$

This means that $x \in A$ and $x \in C$ by the definition of intersection.
$x \in A$ by conjunctive simplification.
$x \in C$ by conjunctive simplification.

Since $x \in A$ and $A \subseteq B$, then $x \in B$.
$x \in B \wedge x \in C$ by conjunctive addition.
$x \in (B \cap C)$ by definition of intersection.

contradiction because x can't be in $B \cap C$ if $B \cap C = \emptyset$.
Therefore our assumption that $x \in (A \cap C)$ must be false.
Since x was arbitrarily chosen, it must be true that $\forall x \in U, x \notin (A \cap C)$.
Therefore $A \cap C = \emptyset$.