Worksheet 10 solns

1. a. $\lim_{n\to\infty} \frac{(-1)^{n+1}}{3n-1}$ ... alternating, so check abs. val.

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1}}{3n+1} \right| = \lim_{n\to\infty} \frac{1}{3n+1} = \lim_{n\to\infty} \frac{1}{n(3+\frac{1}{n})} = \lim_{n\to\infty} \frac{1}{n} \cdot \frac{1}{3+\frac{1}{n}} = \lim_{n\to\infty} \frac{1}{n} \cdot \lim_{n\to\infty} \frac{1}{3+\frac{1}{n}}$$

$$= 0 \cdot \frac{1}{3} = 0.$$

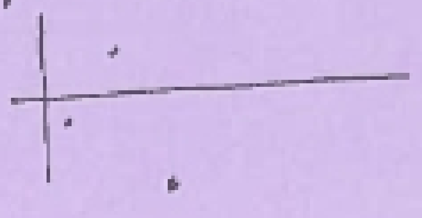
By <u>thm</u> from class, If $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$ So $\boxed{\lim_{n\to\infty} \frac{(-1)^{n+1}}{3n+1} = 0}$

b. $\lim_{n\to\infty} (-2)^n$

$= \lim_{n\to\infty} (-1)^n \cdot 2^n$

alternating, but $\lim_{n\to\infty} |(-1)^n \cdot 2^n| = \lim_{n\to\infty} 2^n = \infty$, so we can't use the theorem.

Observe: $\{-2, 4, -8, 16, -32, 64, \dots\}$



$\boxed{\lim_{n\to\infty} (-2)^n \text{ dNe}}$

c. $\lim_{n\to\infty} \frac{3-2n^2}{3n^3-n+2}$

$= \lim_{n\to\infty} \frac{n^3(\frac{3}{n^3} - \frac{2}{n})}{n^3(3 - \frac{1}{n^2} + \frac{2}{n^3})}$

$= \frac{\lim_{n\to\infty} \frac{3}{n^3} - \lim_{n\to\infty} \frac{2}{n}}{\lim_{n\to\infty} 3 - \lim_{n\to\infty} \frac{1}{n^2} + 2\lim_{n\to\infty} \frac{1}{n^3}}$ $= \frac{0-0}{3-0+0} = \frac{0}{3} = \boxed{0}$

2. a.

$2, 5, 8, 11, 14$ arithmetic $\boxed{a_n = 2+(n-1)3} = 2+3n-3 = 3n-1$

$+3 \quad +3 \quad +3$

$d = 3 \quad a_1 = 2$

b. $1 \quad \frac{-1}{3} \quad \frac{1}{9} \quad -\frac{1}{27} \quad \dots$ $\frac{(-1)^{n+1}}{3^{n-1}}$ or ... geometric

$\times -\frac{1}{3} \quad \times -\frac{1}{3} \quad \times -\frac{1}{3}$

$a_1 = 1, \ r = \frac{-1}{3}$ $\boxed{a_n = \left(\frac{-1}{3}\right)^{n-1}}$

c. $-1 \quad \sqrt{2} \quad -2$ geometric

$\times -\sqrt{2} \quad \times -\sqrt{2}$

$r = -\sqrt{2}, \ a_1 = -1$ $\boxed{a_n = -1(-\sqrt{2})^{n-1}}$

$= (-1)^n (\sqrt{2})^{n-1}$

2d. $54 \quad 18 \quad 6 \quad \ldots$

$\times\frac{1}{3} \quad \times\frac{1}{3}$

geometric

$r=\frac{1}{3} \quad a_1=54$

$\boxed{a_n = 54\left(\frac{1}{3}\right)^{n-1}}$

e. $-3, 2, 7, \ldots$

$+5 \quad +5$

arithmetic

$d=5 \quad a_1=-3$

$\boxed{a_n = -3+(n-1)5}$

$= -3+5n-5$

$= 5n-8$

f. $\frac{16}{27} \quad \frac{8}{9} \quad \frac{4}{3} \quad \frac{2}{1}$

$\times\frac{1/3}{1/3}=\frac{3}{2} \quad \times 3/2 \quad \times 3/2$

$a_n = \frac{2^{5-n}}{3^{4-n}}$ OR geometric

$r=3/2 \quad a_1=\frac{16}{27}$

$\boxed{a_n = \frac{16}{27}\left(3/2\right)^{n-1}}$

same??

let's see: $\frac{16}{27}\cdot\left(\frac{3}{2}\right)^{n-1}$

$= \frac{2^4}{3^3}\cdot\frac{3^{n-1}}{2^{n-1}}$

$= 2^{4-(n-1)}\cdot 3^{n-1-3}$

$= 2^{5-n}\cdot 3^{n-4}$

$= \frac{2^{5-n}}{3^{4-n}}$ ✓ yes they are the same!

3. a. $a_1=6 \quad a_2 \quad a_3 \quad a_4 \quad a_5=66$

$+d \quad +d \quad +d \quad +d$

$6+4d=66$

$4d=60$

$d=15$

$\Rightarrow$

$a_2 = 6+15 = 21$

$a_3 = 36$

$a_4 = 36+15 = 51$

$a_n = 6+(n-1)15$

b. $a_1 \quad a_2 \cdot a_3 \quad a_4 \quad a_5 = 18 \quad a_6 \quad a_7 \quad a_8 \quad a_9 = 30$

$-d \quad -d \quad -d \quad -d \qquad +d \quad +d \quad +d \quad +d$

$18+4d=30$

$4d=12$

$d=3$

$18-4d=a_1$

$18-4(3)=a_1$

$\boxed{6=a_1}$

c. $-4, 4, 12, \ldots$

$+8 \quad +8 \quad +8$

$d=8$

$a_1=-4$

$a_n = -4+(n-1)(8)$

$= -4+8n-8$

$= 8n-12$

what n makes $a_n=92$?

$92 = 8n-12$

$+12 \qquad +12$

$\frac{104}{8} = \frac{8n}{8}$

$\boxed{n=13}$

4. $500 \underset{\times 2}{\frown} 1000 \underset{\times 2}{\frown} 2000$

$\times 2$ every hr

geometric

$r = 2 \quad a_1 = \cancel{500}$ 1000

$b_n = \overset{1000}{\cancel{500}}(2)^{n-1}$

where n = double hrs.

$$\boxed{b_{12} = 1000(2)^{12} \text{ or } 500 \cdot 2^{13} \text{ bacteria}}$$

5. $500, 250, 125, \ldots$

$\div 2 \quad \div 2$

geometric

$r = \frac{1}{2} \quad a_1 = 500$

n = beginning of n^{th} day

$a_n = 500\left(\frac{1}{2}\right)^{n-1}$

$$\boxed{\begin{aligned} a_7 &= 500\left(\tfrac{1}{2}\right)^6 \\ &= \frac{500}{2^6} \text{ mg} \\ &\approx 7.8 \text{ mg} \end{aligned}}$$

6. $180, 360, 540, \ldots$

$+180 \quad +180$

arithmetic

$d = 180, \ a_1 = 180$ — 1 = triangle

2 = square

⋮

10 = 10+2 sides = 12 sides

$$\begin{aligned} a_n &= 180 + (n-1)(180) \\ &= 180 + 180n - 180 \\ &= 180n \end{aligned}$$

$$\boxed{a_{10} = 180(10) = 1800^\circ}$$

7. $a_1 + a_3 = 20$

$a_1 + a_2 + a_3 = 26 \quad \Rightarrow a_2 = 6$

sequence is geometric, so $= \{a_1, \overset{a_2}{\overset{''}{a_1 r}}, \overset{a_3}{\overset{''}{a_1 r^2}}, a_1 r^3, \ldots\}$

so $a_1 r = a_2 = 6$

then $a_1 = \frac{6}{r}$

$a_1 + a_3 = 20$

replace a_3: $a_1 + a_1 r^2 = 20$

factor: $a_1(1 + r^2) = 20$

$a_1 = \frac{20}{1 + r^2}$

$$\begin{aligned} \frac{6}{r} &= \frac{20}{1+r^2} \\ 6(1+r^2) &= 20r \\ 6 + 6r^2 &= 20r \\ 6r^2 - 20r + 6 &= 0 \\ 2(3r^2 - 10r + 3) &= 0 \\ 2(3r-1)(r-3) &= 0 \end{aligned}$$

so $r = 1/3$ or $r = 3$

check if these make sense:

$r=3$: $a_1 3 = 6$

so $a_1 = 2$

know $a_2 = 6$

$a_3 = 6 \cdot 3 = 18$

$+ = 2$

so $r=3$ works w/ $a_1 = 2$

check $r = \frac{1}{3}$: $a_1 \cdot \frac{1}{3} = 6$ so $a_1 = 18$

$a_2 = 6$

$a_3 = 6 \cdot \frac{1}{3} = 2$

$+ = 20$ so works!

two possible answers

$$\boxed{\begin{aligned} a_n &= 2(3)^{n-1} \\ a_n &= 18\left(\tfrac{1}{3}\right)^{n-1} \end{aligned}}$$