Wksht 12 solns

1. a. $g(x) = -2x^2 + 8x - 6$

$\lim_{x\to\infty} g(x) = \lim_{x\to\infty} -2x^2 = \boxed{-\infty} = -(+\text{huge})$

$\lim_{x\to-\infty} g(x) = \lim_{x\to-\infty} -2x^2 = \boxed{-\infty} = -(+\text{huge})$

vertex: $-2(x^2 - 4x \quad) - 6$

$= -2(x^2 - 4x + 4 - 4) - 6$

$= -2(x^2 - 4x + 4) + 8 - 6$

$= -2(x-2)^2 + 2$

vertex: $(2,2)$

x-ints: $-2(x^2 - 4x + 3) = 0$

$-2(x-3)(x-1) = 0$

$x = 3 \quad x = 1$

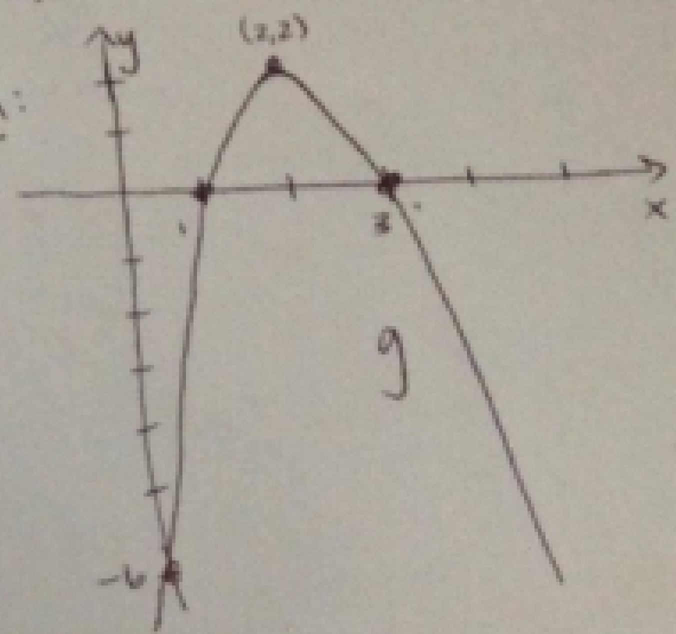
y-int: $g(0) = -6$

sign-chart:

lim		1		3		lim
⊖		z	⊕	z		⊖

$g(2) = - - +$

Sketch:



b. $h(x) = x^3 - 4x^2 - 16x + 64$

$\lim_{x\to\infty} h = \lim_{x\to\infty} x^3 = \infty$

$\lim_{x\to-\infty} h = \lim_{x\to-\infty} x^3 = -\infty$

x-ints: factor by grouping

$x^2(x-4) - 16(x-4) = 0$

$(x-4)(x^2-16) = 0$

$(x-4)(x-4)(x+4) = 0$

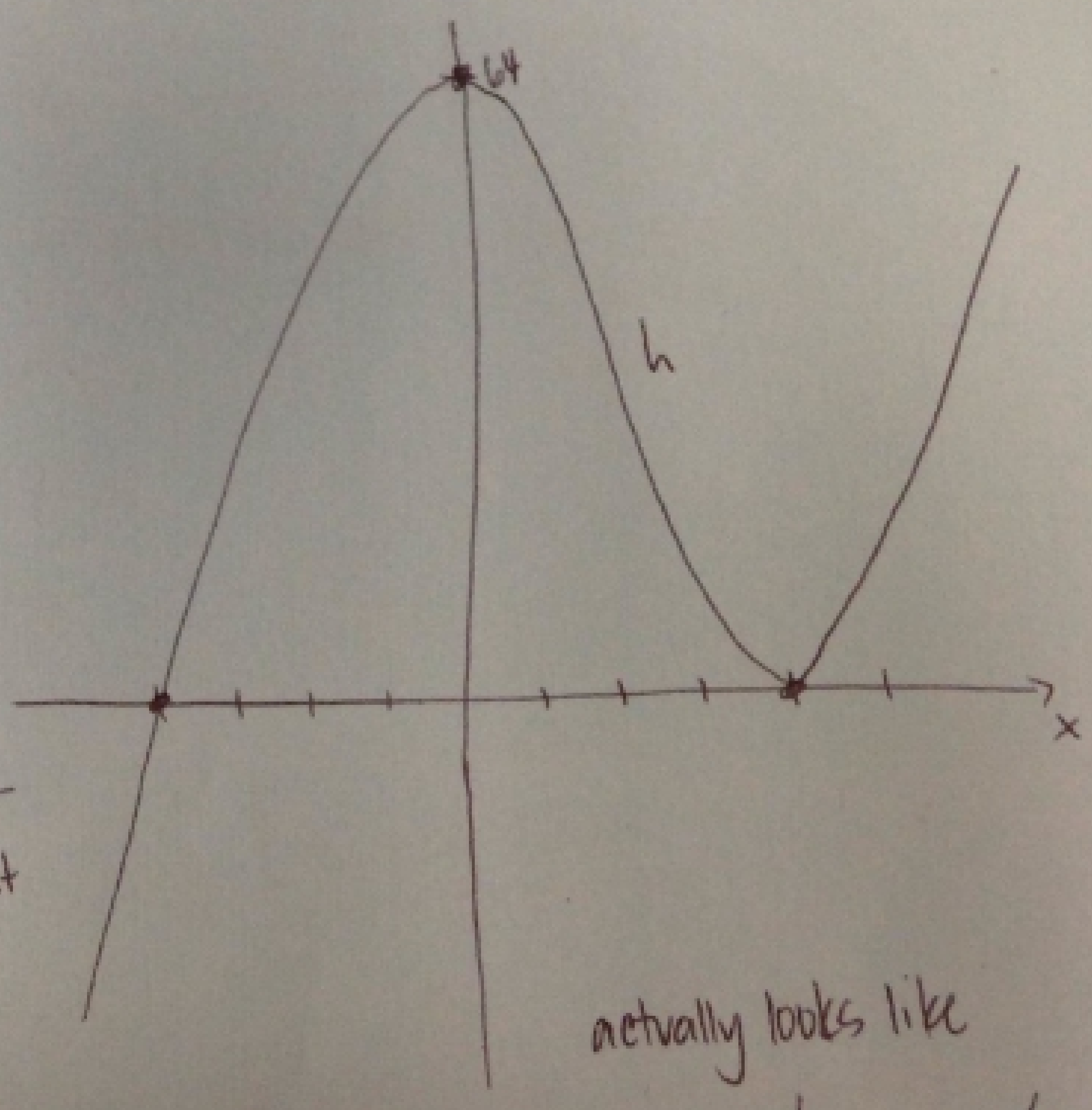
$x = 4 \quad x = -4$

y-int: $h(0) = 64$

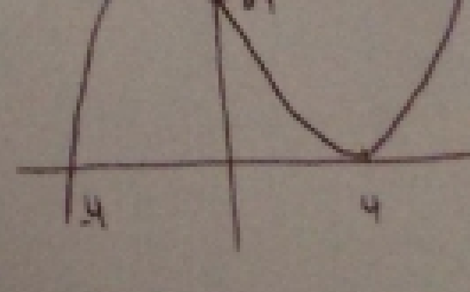
sign chart:

limit		-4		4		limit
⊖		z	⊕	z		⊕

$h(0) = +$



actually looks like



2. $x = \frac{1}{2}$ solution
$\Rightarrow 2x-1$ is factor

$$\begin{array}{r} x^3 - x^2 - 10x - 8 \\ 2x-1 \overline{\smash{)}\, 2x^4 - 3x^3 - 19x^2 - 6x + 8} \\ \underline{-2x^4 + x^3} \\ -2x^3 - 19x^2 - 6x + 8 \\ \underline{+2x^3 \mp x^2} \\ -20x^2 - 6x + 8 \\ \underline{+20x^2 \mp 10x} \\ -16x + 8 \\ \underline{+16x \mp 8} \\ 0 \end{array}$$

$\leftarrow$ remainder 0

so $2x^4 - 3x^3 - 19x^2 - 6x + 8$
$= (2x-1)(x^3 - x^2 - 10x - 8)$

$x = 4$ solution
then $x-4$ factor of $x^3 - x^2 - 10x - 8$

$$\begin{array}{r} x^2 + 3x + 2 \\ x-4 \overline{\smash{)}\, x^3 - x^2 - 10x - 8} \\ \underline{-x^3 + 4x^2} \\ 3x^2 - 10x - 8 \\ \underline{-3x^2 + 12x} \\ 2x - 8 \\ \underline{-2x + 8} \\ 0 \end{array}$$

so
$x^3 - x^2 - 10x - 8$
$= (x-4)(x^2 + 3x + 2)$

$\Rightarrow 2x^4 - 3x^3 - 19x^2 - 6x + 8 = (2x-1)(x-4)(x^2+3x+2)$
$\boxed{= (2x-1)(x-4)(x+2)(x+1)}$ factored form

~~other~~ all solutions: $\boxed{\frac{1}{2}, 4, -2, -1}$

3. $C(x) = \frac{1}{4}x^2 - 6x + 276$ minimize cost? find minimum of parabola... vertex.

$= \frac{1}{4}(x^2 - 24x \quad) + 276$
$= \frac{1}{4}(x^2 - 24x + 144 - 144) + 276$
$= \frac{1}{4}(x-12)^2 - \frac{144}{4} + 276$

$$\begin{array}{r} 276 \\ \underline{-36} \\ 240 \end{array}$$

$= \frac{1}{4}(x-12)^2 + 240$

vertex at $(12, 240)$ so produce $\boxed{\text{12 per hour}}$

2 positive real #s $= x, y$

$x + y = 110$ maximize $x \cdot y$
$= x(110 - x)$
$= -x^2 + 110x$

parabola: find vertex

$-(x^2 - 110x \quad)$
$= -(x^2 - 110x + 55^2 - 55^2)$
$= -(x-55)^2 + 55^2$
$(55, 55^2)$
$\boxed{x = 55,\ y = 55}$

5. a. $2x^3 - 11x^2 + 4x + 5$

try $x=1$: possible rational roots $= \pm\frac{5}{1}, \pm\frac{1}{1}, \pm\frac{5}{2}, \pm\frac{1}{2}$

$$
\begin{array}{r}
2x^2 - 9x - 5 \\
x-1 \overline{\big)\, 2x^3 - 11x^2 + 4x + 5} \\
-2x^3 + 2x^2 \\
\hline
-9x^2 + 4x + 5 \\
+9x^2 - 9x \\
\hline
-5x + 5 \\
+5x - 5 \\
\hline
0
\end{array}
$$

factor $2x^2 - 9x - 5$?

$(2x+1)(x-5)$

$x = -1/2$, $x = 5$

$\boxed{x = 1, 5, -1/2}$

b. $x^4 - x^3 + x^2 - 3x - 6$ possible rational roots $= \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{6}{1}$

$q(1) = 1 - 1 + 1 - 3 - 6 \neq 0$

$q(-1) = 1 + 1 + 1 + 3 - 6 = 0 \Rightarrow x = -1$ soln

so $x+1$ factor

$$
\begin{array}{r}
x^3 - 2x^2 + 3x - 6 \\
x+1 \overline{\big)\, x^4 - x^3 + x^2 - 3x - 6} \\
-x^4 - x^3 \\
\hline
-2x^3 + x^2 - 3x - 6 \\
+2x^3 + 2x^2 \\
\hline
3x^2 - 3x - 6 \\
-3x^2 - 3x \\
\hline
-6x - 6 \\
+6x + 6 \\
\hline
0
\end{array}
$$

$q(x) = (x^3 - 2x^2 + 3x - 6)(x+1)$

$= [x^2(x-2) + 3(x-2)](x+1)$

$= (x^2+3)(x-2)(x+1)$

$x^2 + 3 = 0$

$x^2 = -3$

not real solutions.

$\boxed{x = 2, -1}$

c. $p(x) = x^4 - 4x^2 + 4$ possible rational roots $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}$

$p(1) = 1 - 4 + 4 \neq 0$ $p(2) = 16 - 16 + 4 \neq 0$ $p(4) = 4^4 - 4^3 + 4 = 4(4^3 - 16 + 1) = 4(64 - 15) \neq 0$

$p(-1) = 1 - 4 + 4 \neq 0$ $p(-2) = 16 - 16 + 4 \neq 0$ $p(-4) \neq 0 \ldots$

try factoring... quadratic in disguise!

$u = x^2$ $p(x) = u^2 - 4u + 4$

$= (u-2)(u-2)$

$= (x^2-2)(x^2-2)$

$\Rightarrow x^2 - 2 = 0$

$x^2 = 2$

$\boxed{x = \pm\sqrt{2}}$

not rational, but still real