Final Review

A mass that weighs 5 pounds elongates a spring 6 inch. it is also connected to damping fluid that exerts 3 lbs of force when the velocity is 6 ft/s. if you pull this mass down an additional 6 in write the position functions

$k = \frac{5 lb}{1/2}$: $k = 10$ $\quad$ $5/32 = m$ $\quad$ $c = \frac{3 lb}{6 ft} =$

$my'' + ky$ $\quad$ $0.156y'' + 0.5y' + 10y = 0$

$\frac{1}{0.156}(0.156r^2 + 0.5r + 10 = 0)$ $\quad$ $r^2 + 3.2r + 64 = 0$

$r = \frac{-3.2 \pm \sqrt{3.2^2 - 4(64)}}{2}$

$r = \frac{-3.2 \pm i\sqrt{245.76}}{2}$

$y = e^{-1.6x}(C_1 \cos 7.83x + C_2 \sin 7.83x)$

$r = -3.2 \pm i\, 15.68$

$r = -1.6 \pm i\, 7.83$

$y' = -1.6e^{-1.6x}(C_1 \cos 7.83x + C_2 \sin 7.83x) +$
$e^{-1.6x}(-7.83 C_1 \sin 7.83x + 7.83 C_2 \cos 7.83x)$

$y(0) = 1/2$

$y'(0) = 0$ $\quad$ $0 = -1.6(C_1) + 1(7.83C_2)$

$1/2 = (C_1)$ $\quad$ $Ae^{-3t}\cos(\omega t - \alpha)$

$-0.8 + 7.83C_2 = 0$ $\quad$ $C_2 = 0.102$

$y = e^{-1.6x}(1/2 \cos 7.83x + 0.102 \sin 7.83x)$

$y''' - 4y'' + 14y' - 20y = 0$ $\quad$ $\frac{20}{1}$ $\quad$ $\frac{p}{q} = \frac{1, 20, 2, 10, 4, 5}{1}$ ②

$r^3 - 4r^2 + 14r - 20 = 0$

$r(r^2 - 4r + 14) = 20$

$r = 2$ long division: $r^2 - 2r + 10$ into $r^3 - 4r^2 + 14r - 20 = 0$
$r^3 - 2r^2$ ... $+14r - 20$
$-2r^2 + 4r$ ... $- 20$
$10r - 20$
$10r - 20$
0

$r = \frac{2 \pm \sqrt{4 - 4(1)(10)}}{2}$

$\frac{2 \pm 6i}{2}$

$= 1 \pm 3i$

$y = C_1 e^{2t} + e^{t}(C_2 \cos 3t + C_3 \sin 3t)$

$r^5 - 32 = 0$ $\quad$ $32^{1/5} = (32 + 0i)^{1/5}$ $\quad$ $R = 2$ $\theta = 0 \Rightarrow 2(\cos\frac{0}{5} + i\sin\frac{0}{5})$

$r^5 = 32$ $\quad$ $2(\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5})$

$r = 2$ $\quad$ $2(0.31 + 0.95i)$

$2(-0.8 + 0.6i)$

$2(0.31 - 0.95i)$

$2(-0.8 - 0.6i)$

Undetermine Coeff

$y''' - 6y'' + 11y' - 6y = 3e^{4T}$

$r^3 - 6r^2 + 11r - 6 = 0$ $\frac{6}{1}$ $\frac{1\,2\,3\,6}{1}$

$r-1 \overline{) r^3 - 6r^2 + 11r - 6}$ quotient $r^2 - 5r + 6$

$-r^3 + r^2$

$-5r^2 + 11r - 6$

$+5r^2 - 5r$

$6r - 6$

$-6r + 6$

$r^2 - 5r + 6$

$(r-2)(r-3)$

$C_1e^{T} + C_2e^{2T} + C_3e^{3T} + Y$

$Y = Ae^{4T}$

$Y' = 4Ae^{4T}$

$Y'' = 16Ae^{4T}$

$Y''' = 64Ae^{4T}$

$64Ae^{4T} - 96Ae^{4T} + 44Ae^{4T} - 6Ae^{4T} = 3e^{4T}$

$(-32 + 38)A = 3$ $A = 1/2$

$6A = 3$

$C_1e^{T} + C_2e^{2T} + C_3e^{3T} + 1/2\, e^{4T}$

Variation of Parameters

$y = e^{T}$ $y_2 = e^{2T}$ $y_3 = e^{3T}$

$y' = e^{T}$ $y_2' = 2e^{2T}$ $y_3' = 3e^{3T}$

$y'' = e^{T}$ $y_2'' = 4e^{2T}$ $y_3'' = 9e^{3T}$

$e^{T}\begin{vmatrix} 2e^{2T} & 3e^{3T} \\ 4e^{2T} & 9e^{3T} \end{vmatrix} - e^{T}\begin{vmatrix} e^{2T} & e^{3T} \\ 4e^{2T} & 9e^{3T} \end{vmatrix} + e^{T}\begin{vmatrix} e^{2T} & e^{3T} \\ 2e^{2T} & 3e^{3T} \end{vmatrix}$

$e^{T}(6e^{5T}) - e^{T}(5e^{5T}) + e^{T}(e^{5T}) = 2e^{6T}$

$U_1 = \int \frac{W_1\, 3e^{4T}}{2e^{6T}} = \int \frac{e^{5T}\, 3e^{4T}}{2e^{6T}} = \int 3/2\, e^{3T} = 1/2\, e^{3T}$

$U_2 = -\int \frac{W_2\, 3e^{4T}}{2e^{6T}} = -\int \frac{2e^{4T}\, 3e^{4T}}{2e^{6T}} = -\int 3e^{2T}$

$U_3 = \int \frac{W_3\, 3e^{4T}}{2e^{6T}} = \int \frac{e^{3T}\, 3e^{4T}}{2e^{6T}} = \int 3/2\, e^{T} = 3/2\, e^{T}$

$= 1/s$

$= \frac{1}{s-1}$

$= \frac{1}{s-a}$

$= \frac{n!}{s^{n+1}}$

$F(s) = \mathcal{L}f(t) = \int_0^\infty e^{-st} f(t)\, dt$

$y'' - 4y' + 5y = 3$ $y(0) = 2$ $y'(0) = 1$

$s^2\mathcal{L}_y - sy(0) - y'(0) \quad - 4(s\mathcal{L}_y - y(0)) \quad + 5\mathcal{L}_y$

$(s^2 - 4s + 5)\mathcal{L}_y - 2s - 1 - 8 = \frac{3}{s}$

$\mathcal{L}_y(s^2 - 4s + 5) = 3/s + 9 - 2s$

$\mathcal{L}_y = 3\left(\frac{1}{s(s^2-4s+5)}\right) + 9\left(\frac{1}{s^2-4s+5}\right) - 2\left(\frac{s-2+2}{s^2-4s+5}\right)$

$e^{2T}\cos(T) + 1/2\, e^{2T}\sin(T)$

$\frac{a}{s} + \frac{bs + c}{s^2 - 4s + 5}$

$as^2 - 4as + 5a + bs^2 + cs = 0$

$a + b = 0$ $a = 3/5$

$-4as + cs = 0$ $b = -3/5$

$5a = 3$ $c = 12/5$

$y(0), y'(0) = 0$

$y'' - 5y' + 6y = f(t) \quad f(t) = \begin{cases} 3 & \text{if } t < 2 \\ 5 & \text{if } 2 < t < 6 \\ t & \text{if } t > 6 \end{cases}$

$y'' - 5y + 6y = 3 + 2U_2(t) + (t-5)U_6(t)$

$\mathcal{L}y(s^2 - 5s + 6) = \frac{3}{s} + \frac{2e^{-2t}}{s} + e^{-6s}\left(\frac{1}{s^2} + \frac{1}{s}\right)$

$3\left(\frac{1}{s(s-2)(s-3)}\right) + 2e^{-2t}\left(\frac{1}{s(s-2)(s-3)}\right) + e^{-6s}\left(\frac{1}{s^2(s-2)(s-3)} + \frac{1}{s(s-2)(s-3)}\right)$

$\frac{a}{s} + \frac{b}{s-2} + \frac{c}{s-3} = 1 \quad as^2 - 5sa + 6a + bs^2 - 3sb + cs^2 - 2sc = 1$

$a + b + c = 0 \quad a = 1/6 \quad b + c = -1/6 \quad c = -1/6 - b$

$-5a - 3b - 2c = 0$

$6a = 1 \quad -5/6 - 3b + 2/6 - 2b = -5b = 3/6 = b = -1/30$

$a = 1/6 \; b = -1/2 \; c = 1/3$

$3\left(\frac{1}{6}\left(\frac{1}{s}\right) - \frac{1}{2}\left(\frac{1}{s-2}\right) + \frac{1}{3}\left(\frac{1}{s-3}\right)\right)$

$3\left(\frac{1}{6} - \frac{1}{2}e^{2t} + \frac{1}{3}e^{3t}\right) + 2U_2(t)\left(\frac{1}{6} - \frac{1}{2}e^{2(t-2)} + \frac{1}{3}e^{3(t-2)}\right) + U_6(t)\left(\frac{1}{6} - \frac{1}{2}e^{2(t-6)} + \frac{1}{3}e^{3(t-6)}\right)$

$\frac{as+b}{s^2} + \frac{c}{s-2} + \frac{d}{s-3} \quad as^3 + bs^2 - 5as^2 - 5bs + 6as + 6b + cs^3 - 3cs^2 + ds^3 - 2ds^2$

$a + c + d = 0 \quad b = 1/6$

$1/6 - 25/36 - 3c - 2d = 0 \quad b - 5a - 3c - 2d = 0 \quad a = 5/36$

$\mathcal{L}U_c(t) f(t-c) = e^{-cs}\mathcal{L}f(t) \quad 6 - 25 - 108c - 72d = 0 \quad -5b + 6a = 0 \quad -5/6 + 6a = 0 \quad c = 1/18$

$-108c - 72d = 19 \quad 6b = 1 \quad D = -2/9$

$3\left(\frac{1}{6} - \frac{1}{2}e^{2t} + \frac{1}{3}e^{3t}\right) + 2U_2(t)\left(\frac{1}{6} - \frac{1}{2}e^{2(t-2)} + \frac{1}{3}e^{3(t-2)}\right) + U_6(t)\left(\frac{1}{6} - \frac{1}{2}e^{2(t-6)} + \frac{1}{3}e^{3(t-6)}\right) U_6(t)\left(\frac{5}{36} + \frac{1}{6}(t-6) + \frac{1}{12}e^{2(t-6)} - \frac{2}{9}e^{3(t-6)}\right)$

$y'' - 4y = \delta(t-2)e^{t} \quad \mathcal{L}y \quad \mathcal{L}\delta(t-t_0)f(t) = e^{-st_0}f(t_0)$

$(s^2 - 4)\mathcal{L}y = e^{-2s}e^{2}$

$e^{-2s}e^{2}\left(\frac{1}{s^2-4}\right) \quad \mathcal{L}y = e^2 U_2 \; \frac{1}{2}(\sinh(2(t-2))$

$e^{2} \; \frac{1}{2}\sinh(2t)$