

Part 1: Modeling of Dynamic systems. - now to midterm

- Model 1st order systems by derivating their differential equations
- Model 2nd order systems by deriving their differential equations
- Model these systems using Transfer Functions
- Model these systems using State Space
- Use a model to predict response (transient & steady state)

What is a dynamic system?

- System

• inputs

• outputs

- Dynamic System: there is a delay between input & output

- Mathematical models of dynamic Systems

- Assumptions

- Limitations

- Validity of the model depends on validity of its assumptions

- Differential Equations: main way of modeling dynamic systems

- Feedback control

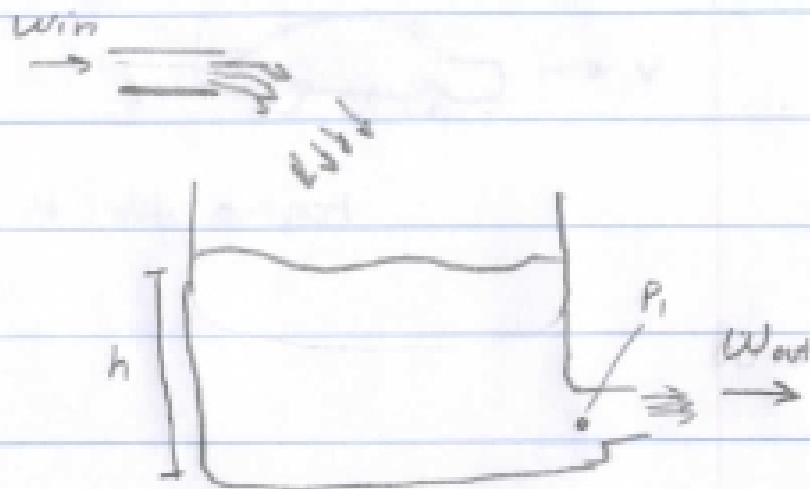
- sensing + computation + actuation

1st order dynamic system

- 1st order: system has 1 state variable

- 1st order: differential equation of system is order 1

- Example 1: a fluid tank



h - know $h(t) = ?$

- W_{in}
- W_{out}
- A
- Open/closed system
- $h(0)$
- ↳ pressure
- \sqrt{h}

Conservation of mass → could be considered conservation of volume

$$\frac{dV}{dt} = W_{in} - W_{out}$$

$V = A \cdot h$ $W_{out} = c h$ → a constant defining or face

$$A \frac{dh}{dt} = W_{in} - W_{out} \rightarrow \underline{A \dot{h} = W_{in} - c h} \rightarrow \text{1st order differential equation or 1st order dynamic system}$$

$$A \dot{h} + c h = W_{in}$$

$h(t) = x$ → state variable. 1 state variable thus 1st order

$W_{in} = U$ → input to the system

$h(t) = y$ → output of the system → need to know

$T(t) = x$

$$m C_p \frac{dT}{dt} = Q_{in} - Q_{out}$$

steady state $T_{ss} = 0$

$$m C_p \dot{T} = Q_{in} - h A (T - T_{ss}) \rightarrow \boxed{m C_p \dot{T} + h A T = Q_{in}}$$

Example 3 of a 1st order system

input = force of engine

output = velocity, $v(t)$



need model to relate the input to the output

↳ write differential equation



Assumptions:

$$v=0 \Rightarrow F_f = 0, \quad F_d = CV$$

$$\frac{d}{dt} mv = \Sigma F \rightarrow m \frac{dv}{dt} = \Sigma F \rightarrow ma = \Sigma F$$

$$m \frac{dv}{dt} = F - CV \rightarrow \boxed{m \dot{v} + CV = F} \quad \text{state variable} = \text{Velocity}$$

Example 4 - Heat Transfer Example

- Assume an insulated space with heater

- $x(t) = T(t)$: State Variable

- $y = T$ → output variable

- $U = Q_{in}$ → input variable

need to know

$T(0), h, A, K, b, C_p$

$$m C_p \frac{dT}{dt} = Q_{in} - Q_{out}$$

Assume:

only convection, $T_{\infty} = 0$

$$m C_p \dot{T} = Q_{in} - hA(T - T_{\infty}) \rightarrow \boxed{m C_p \dot{T} + hAT = Q_{in}}$$