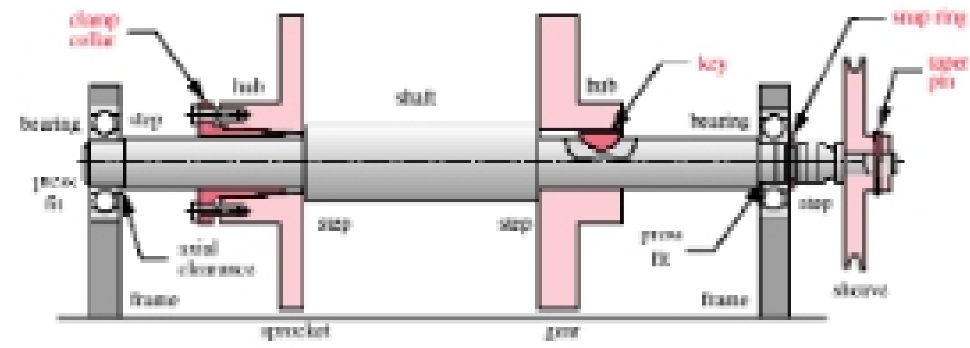
Shafts & Keys

- Shafts in general
- Fatigue
- Deflection
- Keys
- Critical Frequencies

Shafts



shoulders, keys, bending, torsion, deflection

Shaft Power

Power = (Torque)(Angular Velocity)

$$P = T\omega$$

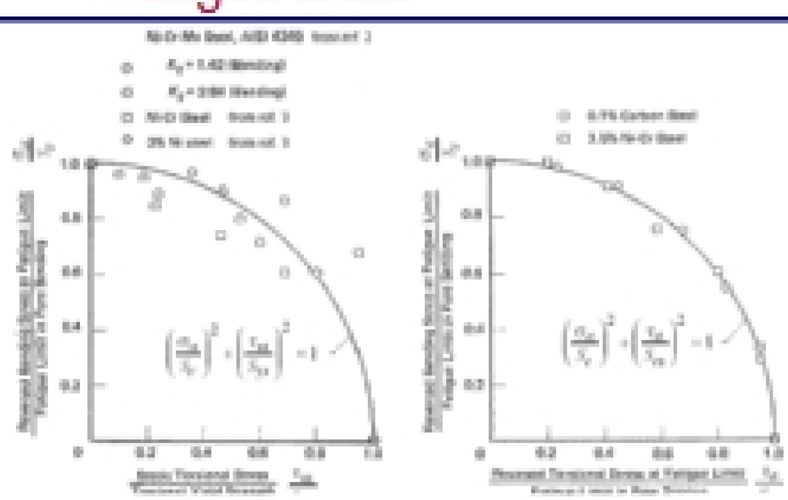
W = (Nm)(radians/s)

1 hp = 745.7 W

Stresses in Shafts

- normal, bending, alternating, σ_a
- normal, bending, mean, σ_m
- shear, torque, alternating, τ_a
- shear, torque, mean, τ_m

Fatigue Data

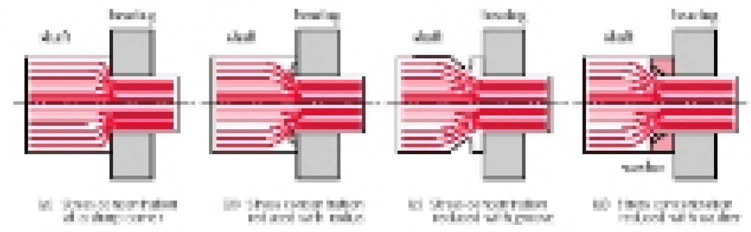


Shaft Design Strategy

- Find correct equation for d_{shaft}
- Identify critical points along shaft
  - Find M_a, M_m, T_a, T_m
  - Find K_f, K_{fs}, K_{fm}, K_{fsm}
- Find S_e (as a function of d_{shaft})
- Solve for d_{shaft} (iterative)

Some General Rules of Shaft Design

- Length/positioning
  - keep length as short as possible, avoid overhangs/cantilevered sections
- Hollow shafts
  - greater stiffness/mass and higher natural frequency (but has greater cost and diameter)
- Stress concentrations
  - place where bending moment is low, use generous radii



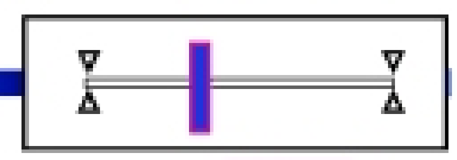
More General Rules

- Gears
  - deflection less than 0.005 in., relative slope differ by less than 0.03 degrees
- Bearings
  - deflection important for sleeve/journal bearings
  - slope important for roller bearings
- Natural Frequency
  - first natural frequency > 3x(forcing frequency)

Key Design Strategy

- For Direct Shear Failure
  - Find F_{key} from $F=T/r_{shaft}$
    - you now have F_a and F_m
  - Find τ_a and τ_b (=F/A_{shear})
  - Find σ'_a and σ'_m

$$\frac{1}{N_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}$$



- For Bearing Failure
  - $F=F_m+F_a$
  - Find σ (=$F/A_{bearing}$)

$$N_s = \frac{S_y}{\sigma}$$

Natural Frequencies

- Bending
  - Lateral
  - Whirl
- Torsional

Lateral

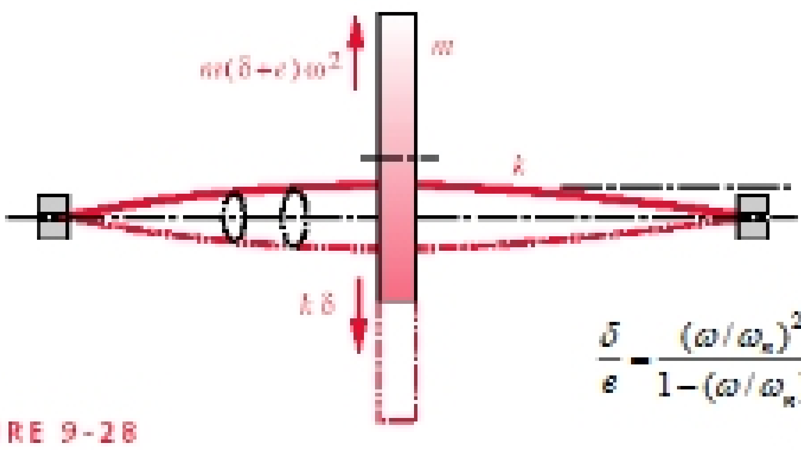
assumes external excitation

set potential energy equal to kinetic energy

$$Natural\ Frequency = \sqrt{gravity \cdot \frac{SUM(weight * deflection)_{at\ each\ weight}}{SUM(weight * deflection^2)_{at\ each\ weight}}}$$

$$\omega_n = \sqrt{g \cdot \frac{\sum_{i=1}^{n} w_i \delta_i}{\sum_{i=1}^{n} w_i \delta_i^2}}$$

Shaft Whirl



$$\frac{\delta}{e} = \frac{(\omega/\omega_n)^2}{1-(\omega/\omega_n)^2}$$

FIGURE 9-28

Shaft Whirl (amplitude greatly exaggerated)

Whirl

$$\frac{\delta}{e} = \frac{(\omega/\omega_n)^2}{1-(\omega/\omega_n)^2}$$

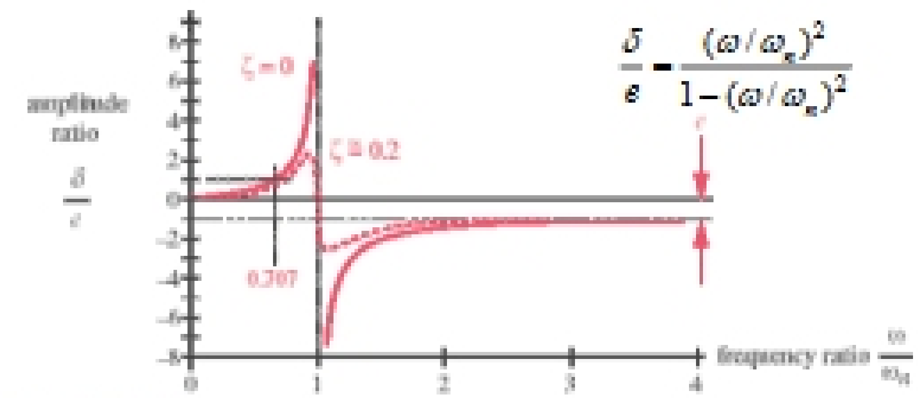
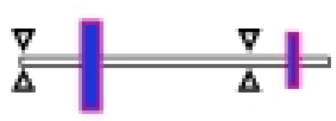


FIGURE 9-29

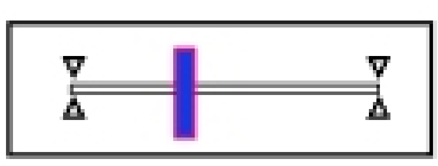
Amplitude Response of a Self-Excited Shaft-Whirl System as a Function of Frequency Ratio

Bending Frequency Strategy

- Find maximum static deflection
  - static, but be realistic



- Find ω_n using Lateral Deflection
- Find ω/ω_n



Torsional Frequency Strategy

- Find I_m of mass (ignore shaft)
- Find K_t
  - Find J for each section
  - $K_{t,section}$=GJ/L
  - Find $1/K_{t,total} = 1/K_1 + 1/K_2 + 1/K_3 + \ldots$
- $\omega_n = \sqrt{\frac{K_t}{I_m}}$