MOTION IN 2-D

POSITION, VELOCITY AND ACCELERATION VECTORS

Position: $\vec{r} = (x, y)$

Velocity: $\vec{v} = (v_x, v_y) = \dfrac{d\vec{r}}{dt} = \left(\dfrac{dx}{dt}, \dfrac{dy}{dt}\right)$

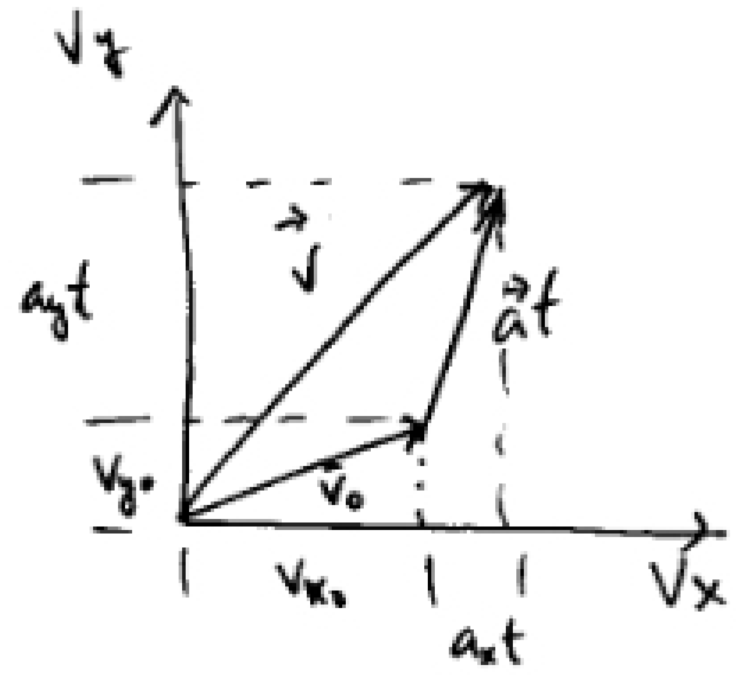
Acceleration: $\vec{a} = (a_x, a_y) = \dfrac{d\vec{v}}{dt} = \left(\dfrac{dv_x}{dt}, \dfrac{dv_y}{dt}\right)$

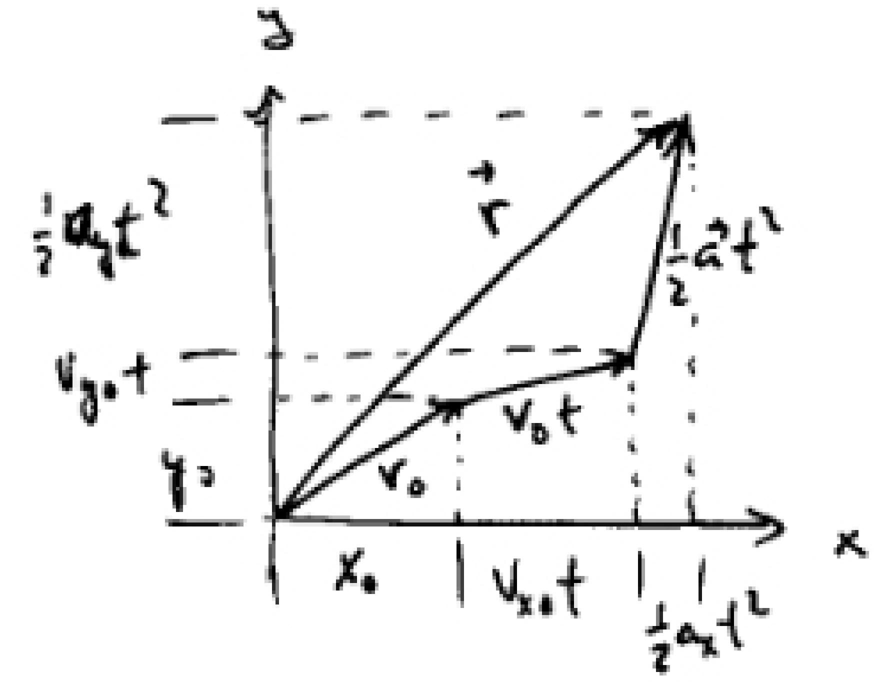
2-D MOTION WITH CONSTANT ACCELERATION

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \quad \begin{cases} x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 \\ y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2 \end{cases}$$

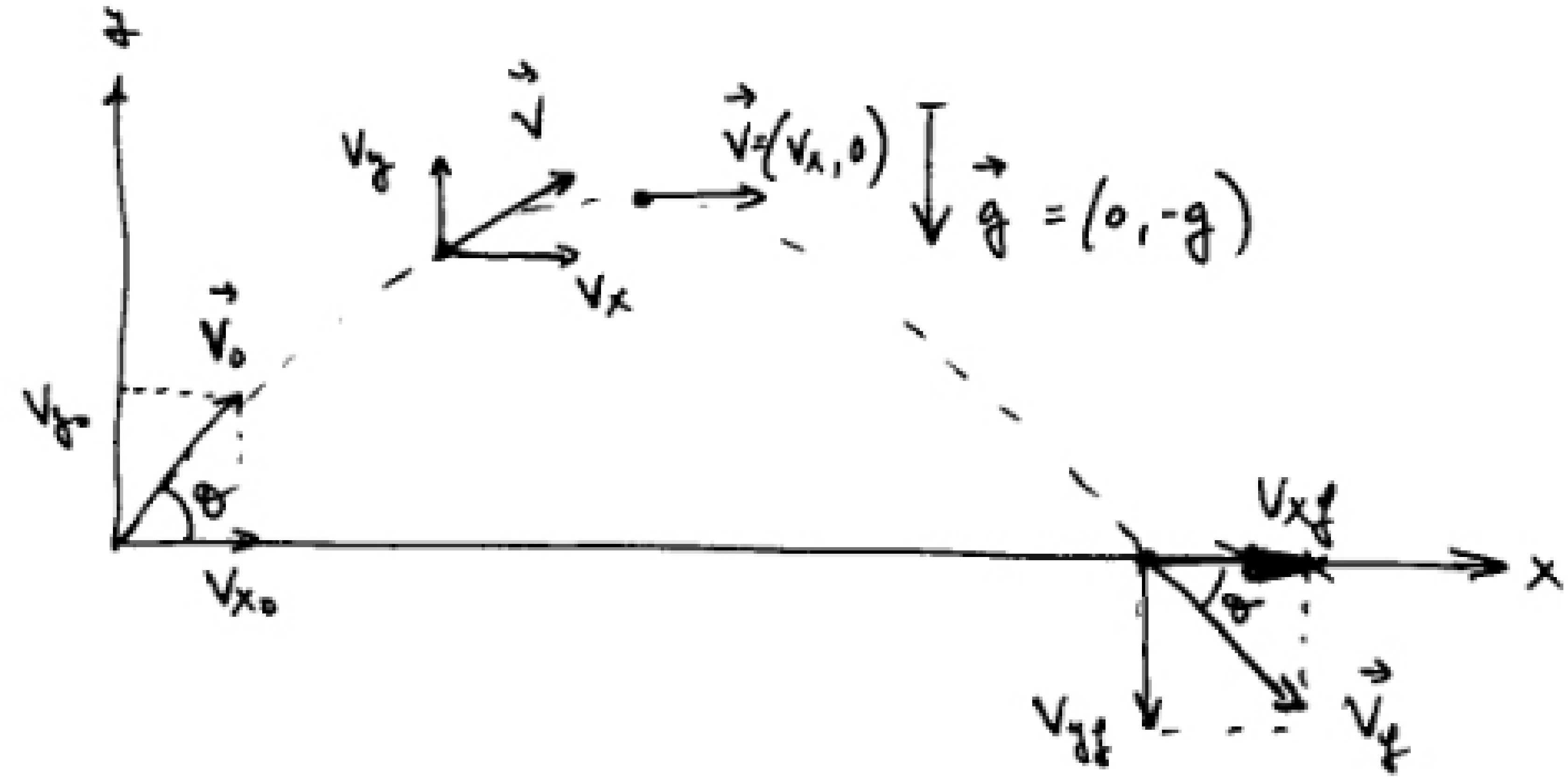
$$\vec{v} = \vec{v}_0 + \vec{a}t \quad \begin{cases} v_x = v_{x0} + a_x t \\ v_y = v_{y0} + a_y t \end{cases}$$

* Vectorial representation of the eq. of motion in 2-D.





PROJECTILE MOTION



$$\left.\begin{aligned}\vec{r} &= \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{g}t^2 \\ \vec{v} &= \vec{v}_0 + \vec{g}t\end{aligned}\right\} \quad \left.\begin{aligned} x &= x_0 + v_{x0}t \\ y &= y_0 + v_{y0}t - \tfrac{1}{2}gt^2 \\ v_x &= v_{x0} \\ v_y &= v_{y0} - gt \end{aligned}\right\} \quad \begin{aligned} v_{x0} &= v_0\cos\theta \\ v_{y0} &= v_0 \sin\theta \end{aligned}$$

We can find a function to describe the trajectory of the projectile from $y(x)$:

$$x = x_0 + v_{x0}t \rightarrow \text{let's make } x_0 = 0 \rightarrow x = v_{x0}t \rightarrow t = \frac{x}{v_{x0}}$$

$$y = y_0 + v_{y0}t - \tfrac{1}{2}gt^2 \rightarrow y_0 = 0 \rightarrow y = v_{y0}t - \tfrac{1}{2}gt^2$$

Therefore: $$y(x) = \left[\tan\theta\right]x - \left[\frac{1}{2}\frac{g}{v_0^2\cos^2\theta}\right]x^2$$

which is the eq. of a parabola.

Maximum height of the projectile

There are two ways to find the maximum height of a projectile.

1) Making $v_y = v_{y0} - gt = 0$, and substituting t_m in $y(t)$.

2) Derivating $y(t) = y_0 + v_{y0} t - \frac{1}{2} g t^2$ and making $\frac{dy}{dt} = 0$

$$\frac{dy}{dt} = 0 = v_{y0} - gt \longrightarrow \boxed{t_m = \frac{v_{y0}}{g} = \frac{v_0 \sin\theta}{g}}$$

time to arrive to a maximum height.

Then, we substitute this time in $y(t)$ and find,

$$y_m = \underbrace{y_0}_{0} + \underbrace{v_{y0}}_{v_0 \sin\theta} \left(\frac{v_0 \sin\theta}{g} \right) - \frac{1}{2} g \left(\frac{v_0 \sin\theta}{g} \right)^2$$

$$\boxed{y_m = h = \frac{v_0^2 \sin^2\theta}{g}}$$ Maximum height