Lab 1: Entanglement and Bell's Inequalities

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Abstract

The purpose of this laboratory is to demonstrate polarization entanglement of a pair of photons created by spontaneous parametric down-conversion in two Type-1 beta barium borate crystals. We showed violation of the CHSH inequality by the number of coincident detections of the signal and idler photons on our two avalanche photo diode detectors using rotating linear polarizers to select 16 different incident polarizations for the purpose of evaluating the CHSH inequality. Achieving a value of $|S| = 2.64$, our results confirm that no local hidden variable theory accounts for the correlation between the photon polarizations, i.e. we successfully demonstrated photon polarization-entanglement.

1 Introduction

Entanglement is a property of multiparty quantum states that are not factorable into a product of states each describing just one party of the multiparty system: $|\psi_{12}\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle$. Entangled states, or non-separable states, have the property that a complete description of any individual component of the system involves a description of the entangled partners as well. One may not completely describe an individual party of an entangled pair, for instance, without reference to the other party. Furthermore, measurement of an entangled party yields information about all parties in the entanglement. Thus, one may gain information about the state of an entangled party nonlocal to a measurement on its entangled partner. Entanglement, and its implied nonlocality, was unsettling to Einstein, Podolsky, and Rosen, who, in their famous EPR paper in 1935 set out to demonstrate how the prediction of entanglement in quantum mechanics was an indicator that the theory was incomplete [1]. However, in 1965, John Bell derived a rule to which any classical hidden variable theory must adhere [2] (see also [3]). He showed that nonlocal theories would violate an inequality that classical theories must obey, thus giving a testable way to determine if a system behaves classically, as EPR insisted it must, or demonstrates a type of quantum nonlocality that we now call entanglement.

In this laboratory we test the violation of the Clauser-Horne-Shimony-Holt version of Bell's inequality using polarization-entangled photons created by spontaneous parametric down-conversion in two orthogonal Type-1 BBO crystals [4]. We follow the experimental procedure developed by Kwiat *et. al.* [5] (see also [6], [7]).

1.1 Theory

We use the Clauser-Horne-Shimony-Holt (CHSH) version of Bell's inequality in this experiment [6]. The SPDC state that we will be measuring has the entangled form: $|\psi_{Bell}\rangle = \frac{1}{\sqrt{2}}(|V\rangle_s|V\rangle_i + |H\rangle_s|H\rangle_i)$, where V/H denote vertical/horizontal polarizations, and s/i indicate the signal/idler photons that result from the SDPC in the Type-I BBOs. We note that our entangled state is invariant under change of polarization basis, and show this by rewriting our state in an arbitrary polarization basis at an angle α to our original horizontal and vertical states. By simplifying, we see that our original state is intact.

$$
\begin{aligned}
\frac{1}{\sqrt{2}}(|V_\alpha\rangle_s|V_\alpha\rangle_i + |H_\alpha\rangle_s|H_\alpha\rangle_i) = \frac{1}{\sqrt{2}}[&(\cos(\alpha)|V\rangle_s + \sin(\alpha)|H\rangle_s)(\cos(\alpha)|V\rangle_i + \sin(\alpha)|H\rangle_i) \\
&+ (-\sin(\alpha)|V\rangle_s + \cos(\alpha)|H\rangle_s)(-\sin(\alpha)|V\rangle_i + \cos(\alpha)|H\rangle_i)] \\
= \frac{1}{\sqrt{2}}[&\cos^2(\alpha)|V\rangle_s|V\rangle_i + \sin^2(\alpha)|H\rangle_s|H\rangle_i \\
&+ \sin(\alpha)\cos(\alpha)|V\rangle_s|H\rangle_i + \sin(\alpha)\cos(\alpha)|H\rangle_s|V\rangle_i \\
&+ \sin^2(\alpha)|V\rangle_s|V\rangle_i + \cos^2(\alpha)|H\rangle_s|H\rangle_i \\
&- \sin(\alpha)\cos(\alpha)|V\rangle_s|H\rangle_i - \sin(\alpha)\cos(\alpha)|H\rangle_s|V\rangle_i] \\
= \frac{1}{\sqrt{2}}&(|V\rangle_s|V\rangle_i + |H\rangle_s|H\rangle_i) \\
= |&\Psi_{Bell}\rangle
\end{aligned}
$$

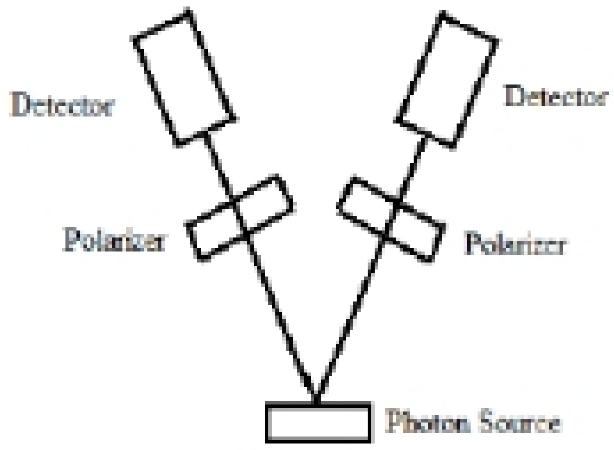

Figure 1: Depiction of polarization-entangled photon pairs passed through polarizers and incident on APDs.

Measurement of the polarization of these photons in any basis thus preserves their entangled state. Indicated in Figure 1, our experiment places APDs so that two detectors sit on diametrically opposed sides of the SPDC cone of entangled light that is emitted from the BBOs. Thus, each APD will receive one of the entangled photons. Placing a polarizer in front of each APD, we can control the measurement basis of each APD, setting one polarizer to an angle α, and the other to an angle β. To evaluate the CHSH inequality we will want the probability that both photons are vertically polarized (both horizontally polarized, and one in each polarization) in their respective polarizer basis. Below we compute this first probability and state the resultant $\cos^2(\alpha-\beta)$ or $\sin^2(\alpha-\beta)$–dependence for the remaining combinations.

$$\begin{aligned}
P_{VV}(\alpha,\beta) &= |\langle V_\alpha|_i \langle V_\beta|_s |\Psi_{Bell}\rangle|^2 \\
&= \frac{1}{2}|[\langle V|_i \cos(\alpha) + \langle H|_i \sin(\alpha)][\langle V|_s \cos(\beta) + \langle H|_s \sin(\beta)][|V\rangle_i|V\rangle_s + |H\rangle_i|H\rangle_s]|^2 \\
&= \frac{1}{2}|\cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)|^2 \\
&= \frac{1}{2}\cos^2(\alpha-\beta).
\end{aligned}$$

The remaining probabilities of polarization combinations follow the same way and are:

$$\begin{aligned}
P_{HH}(\alpha,\beta) &= \frac{1}{2}\cos^2(\alpha-\beta) \\
P_{VH}(\alpha,\beta) &= \frac{1}{2}\sin^2(\alpha-\beta) \\
P_{HV}(\alpha,\beta) &= \frac{1}{2}\sin^2(\alpha-\beta).
\end{aligned}$$

Let $N(\alpha,\beta)$ be the experimentally measured number of coincident signal and idler photon counts for polarization alignments at angles α and β respectively. Then, we may analogously define the above probabilities in terms of these data.

$$\begin{aligned}
P_{VV}(\alpha,\beta) &= \frac{N(\alpha,\beta)}{N_{Total}} \\
P_{HH}(\alpha,\beta) &= \frac{N(\alpha_\perp,\beta_\perp)}{N_{Total}} \\
P_{VH}(\alpha,\beta) &= \frac{N(\alpha,\beta_\perp)}{N_{Total}} \\
P_{HV}(\alpha,\beta) &= \frac{N(\alpha_\perp,\beta)}{N_{Total}}.
\end{aligned}$$

We have introduced the notation $\alpha_\perp = \alpha + 90°$, and $\beta_\perp = \beta + 90°$, and

$N_{Total} = N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) + N(\alpha_\perp,\beta) + N(\alpha,\beta_\perp)$, which is the total number of coincident counts of signal/idler detections. Hence, we can measure the probabilities $P_{HH}, P_{HV}, P_{VH}, P_{VV}$ in our experiment by recording the number of coincident signal/idler photon pairs detected at the APDs when the polarizers are aligned to the different combinations of angles. Using this, we must choose angles for which it is possible to violate Bell's inequality. The CHSH inequality uses a correlation of the probabilities defined as follows:

$$\begin{aligned} E(\alpha,\beta) &= P_{VV} + P_{HH} - P_{VH} - P_{HV} \\ &= \frac{1}{2}\cos^2(\alpha-\beta) + \frac{1}{2}\cos^2(\alpha-\beta) - \frac{1}{2}\sin^2(\alpha-\beta) - \frac{1}{2}\sin^2(\alpha-\beta) \\ &= \cos^2(\alpha-\beta) - \sin^2(\alpha-\beta) \\ &= \cos(2(\alpha-\beta)). \end{aligned}$$

We may also write the probability correlation above in terms of the photon counts using the experimental definition of the coincidence probabilities just defined. Then we have:

$$E(\alpha,\beta) = \frac{N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) - N(\alpha,\beta_\perp) - N(\alpha_\perp,\beta)}{N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) + N(\alpha,\beta_\perp) + N(\alpha_\perp,\beta)}.$$

The Clauser-Horne-Shimony-Holt inequality considers a convex combination of the correlation function above, using the four angle combinations, as displayed here: $S = |E(a,b) - E(a,b')| + |E(a',b) + E(a',b')|$, where S is the quantity that is bounded for a classical system operating according to a local hidden variable theory; we must have $|S| \le 2$. Note, however, that for choices of angle $a = -\pi/4$, $b = -\pi/8$, $a' = 0, b' = \pi/8$, the predicted value of S from the above derivation starting with the rotationally invariant entangled Bell state that describes our setup of signal/idler entangled photon pair is $S = \left|-1/\sqrt{2} - 1/\sqrt{2}\right| + \left|1/\sqrt{2} + 1/\sqrt{2}\right| = 2\sqrt{2}$. So, the quantum mechanical description of our system violates the inequality.

The previous discussion assumes 100% visibility. Visibility is defined as follows:

$\mathit{Visibility} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \xrightarrow{\text{for discrete measurements}} \frac{N_{\max} - N_{\min}}{N_{\max} + N_{\min}}$, where I is intensity, and N is the number of detections made. We compute visibility as a check on our data to ensure that we meet a minimum condition on observation of an entangled state: visibility larger than $1/\sqrt{2}$.

Our goal then is this: we seek to observe a value of S > 2 to suggest entanglement is present in our system. We note also that to violate Bell's Inequality in order to observe entanglement in our system, we should achieve a visibility larger than 0.71, in addition to a value of S > 2 and the $\cos^2(\alpha-\beta)$ dependence of coincident photons [8].