EE 422G - Signals and Systems Laboratory

Lab 1 Sampling and Quantization

Written by
Kevin D. Donohue
Department of Electrical and Computer Engineering
University of Kentucky
Lexington, KY 40506
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Objectives:

- Introduction to Matlab and Simulink features for signal analysis.
- Apply Matlab to examine relationship between quantization bits and signal-to-noise ratios (SNR).
- Apply Simulink to examine aliasing and the impact of non-ideal low-pass filtering for reconstructing a signal from its samples.

1. Back ground

This laboratory exercise focuses on the relationship between noise and interference resulting from digitizing an analog signal. Digitization results in 2 types of noise. The first is from sampling a continuous-time (CT) signal at discrete points in time, which causes ***aliasing***. If the signal is not sampled at a rate higher than twice its highest frequency, then interference from aliasing may occur. The second type of noise results from rounding-off the sample amplitudes to discrete levels. This rounding error results in an additive noise process referred to as ***quantization noise***.

Sampling and Aliasing:

A mathematical representation of the sampling operation is given by a sequence of impulse functions in continuous time:

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT), \qquad (1)$$

where the impulses are separated by T seconds (sampling interval) corresponding to a sampling frequency of $F_s = \frac{1}{T}$ Hz. The sampling function $s(t)$ is shown in Fig. 1.

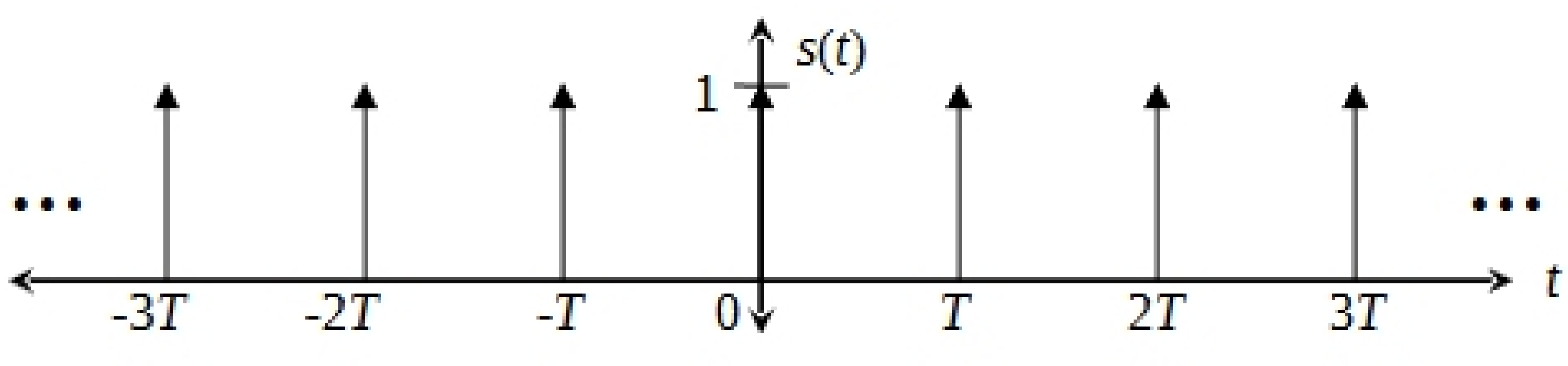

Figure 1. Impulse train to model sampling operation.

It can be shown that the Fourier Transform of $s(t)$ is also an impulse train in the frequency domain with impulses separated by the sampling frequency:

$$\hat{S}(f) = \frac{1}{T}\sum_{k=-\infty}^{\infty}\delta(f - kF_s) \qquad (2)$$

This impulse train is illustrated in Fig. 2.

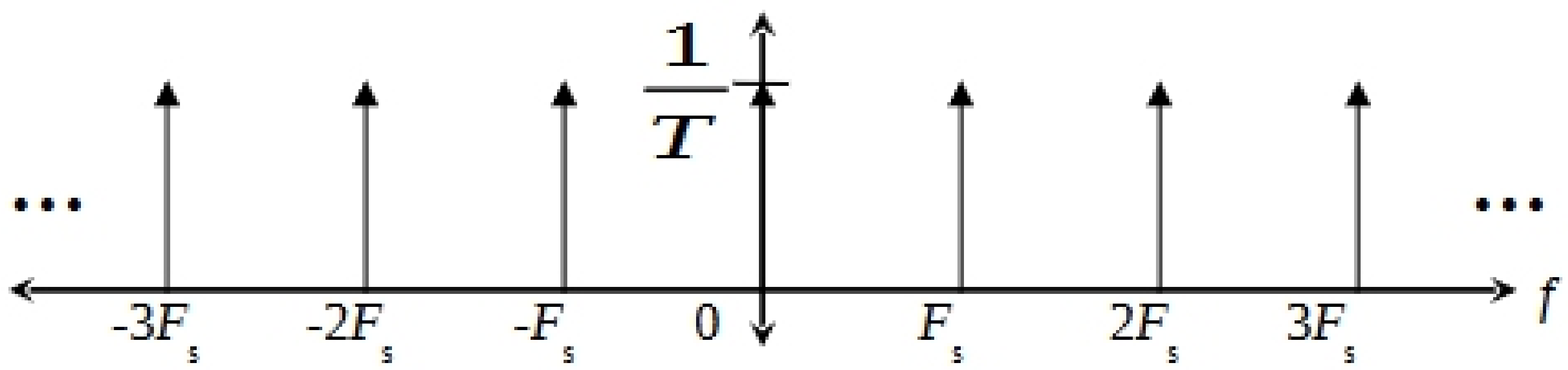

Figure 2. Fourier transform of sampling impulse train that creates aliased spectra.

Now consider a CT signal, $x_c(t)$, that is bandlimited to F_N :

$$X_c(f) = 0 \quad \text{for} \quad |f| > F_N \qquad (3)$$

where F_N is referred to as the bandlimit. As an example, consider the Fourier Transform of bandlimited signal presented in Fig. 3.

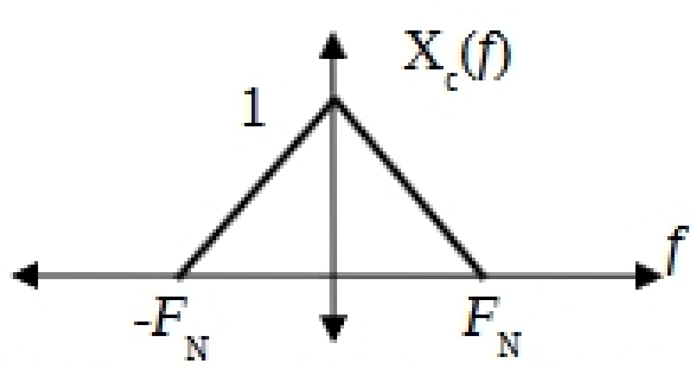

Figure 3. Example of Fourier Transform of a bandlimited continuous-time signal.

Consider sampling $x_c(t)$ by multiplying it with the impulse train of Eq. (1), which effectively zeros out the information between sampling points:

$$x_s(t) = x_c(t)\sum_{n=-\infty}^{\infty}\delta(t - nT)\,. \qquad (4)$$

Now for a linear time-invariant (LTI) system, multiplication in the time domain corresponds to convolution in the frequency domain, given by:

$$\hat{X}_s(f) = \hat{X}_c(f) * \frac{1}{T}\sum_{k=-\infty}^{\infty}\delta(f - kF_s) = \frac{1}{T}\int_{-\infty}^{\infty}\hat{X}_c(f-\lambda)\sum_{k=-\infty}^{\infty}\delta(\lambda - kF_s)\,d\lambda\,. \qquad (5)$$

Since the convolution of a signal with a shifted delta Dirac function, results in a shift version of the signal, Equation (5) shows that sampling replicates the original signal spectrum along the frequency axis separated by integer multiples of the sampling frequency F_s. The convolution integral of Eq. (5) becomes:

$$\hat{X}_s(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} \hat{X}_c(f - kF_s) \qquad (6)$$

The aliasing described in Eq. (6) is illustrated in Fig. 4 for two cases. Figure 4a shows the case when the sampling frequency is greater than twice F_N . Figure 4b shows the case when the sampling frequency is less than twice F_N .

The Nyquist frequency defined as half of the sampling frequency of a digital processing system. This is also referred to as the folding frequency since frequencies beyond this value fold back onto the non-aliased spectral range. Note that when the conditions on the sampling theorem are met (i.e. Nyquist frequency is greater than F_N), the aliased spectra do not interfere with each other and the original signal can be recovered from its samples with an idea low-pass filter. When overlap between aliased spectra occurs, the interference results in an irreversible noise/distortion process and the original signal cannot be recovered. Aliasing in this case irreversibly degrades the signal. Bandlimit F_N is a property of the continuous-time signal and is sometimes referred to as the Nyquist rate. Note: The Nyquist frequency is a property of the processing system while Nyquist rate is a property of the continuous-time signal.

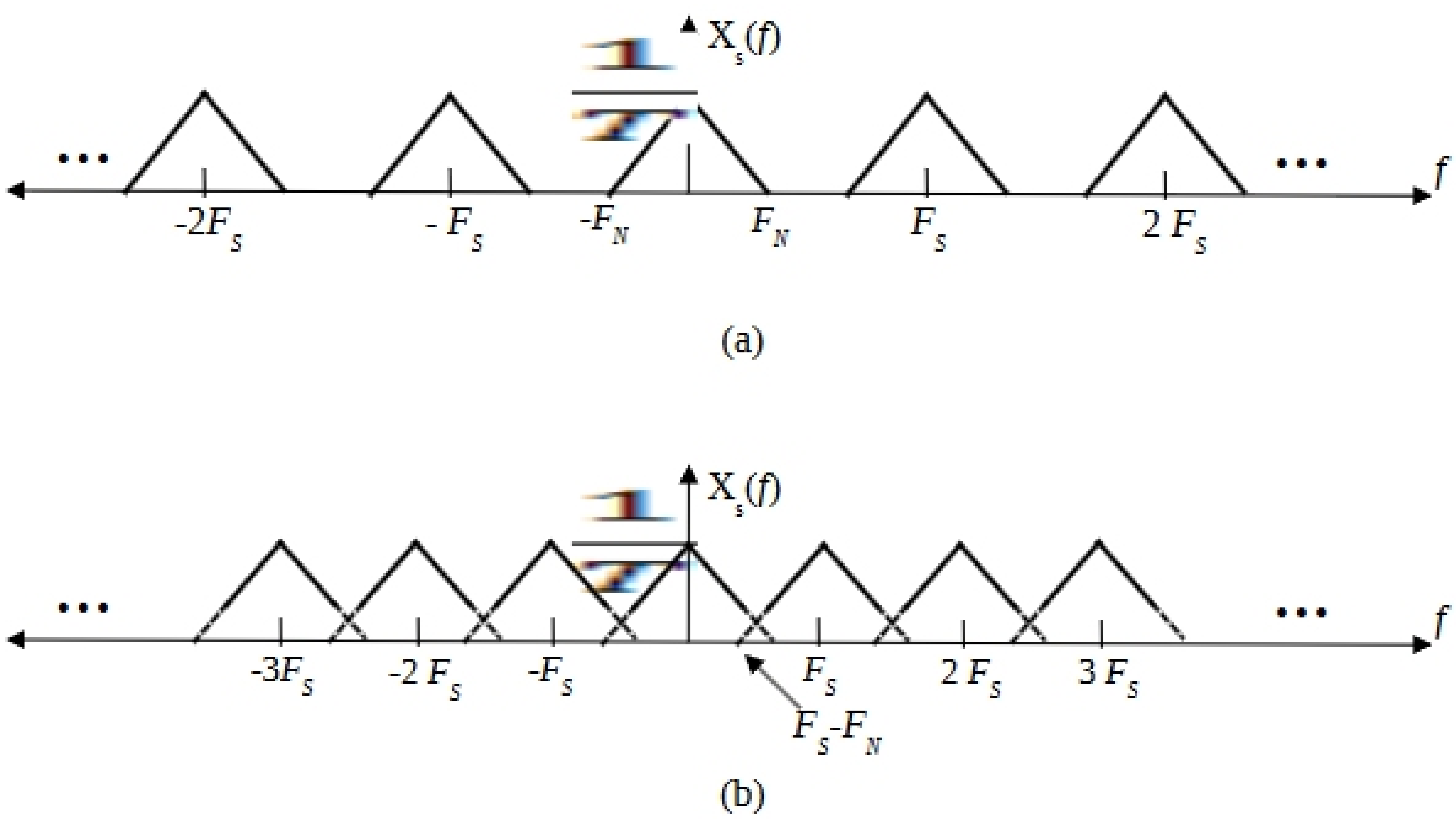

Figure 4. Aliased spectra when sampling rate is (a) greater than twice the bandlimit; $F_s > 2F_N$, (b) less than twice the bandlimit, $F_s < 2F_N$.

The derivation leading to Fig. 4 is the basis for the sampling theorem, which states that a bandlimited signal can be recovered from it samples if it is sampled at a rate greater than