University of Kentucky

EE 422G - Signals and Systems Laboratory

Lab 3 - IIR Filters

Objectives:

- Use filter design and analysis tools to create IIR filters based on general filter specification.
- Understand the impact of different computational structures for filter implementation
- Understand the impact of the placement of poles and zeros on the frequency response of the filter.

1. Background

Infinite impulse response (IIR) filters are an alternative to finite impulse response (FIR) filters. Often an IIR implementation can meet a given filter specification with less computation than an FIR implementation, but IIR filters induce nonlinear phase, can be potentially unstable, and are more sensitive to numerical problems.

Like FIR filters, IIR filters are linear time-invariant (LTI) systems that can recreate a wide range of frequency responses. IIR implementations with specified stopband-attenuation and transition-band requirements, typically requires far fewer filter taps than an FIR filter meeting the same specifications. This leads to a significant reduction in the computational complexity required to achieve a given frequency response. However, IIR filters have poles, which require feedback for implement and can result in instability if not properly placed inside the unit circle. The feedback can increase the sensitivity to errors introduced during computations, especially for fixed-point processors. In addition, IIR filters result in nonlinear phase distortion (delaying different frequency components of the input signals by different amounts). Some of these complications are explored in this lab.

Up to this point most of your filter implementations have been in terms of "Direct Form," which is suggested by converting the transfer function directly to a difference equation:

$$H(z) = \frac{\sum_{m=0}^{M} b_m z^{-m}}{1 + \sum_{n=1}^{N} a_n z^{-n}} \tag{1}$$

However, if the polynomials are factored into second order stages as such:

$$H(z) = G_0 \prod_{p=1}^{P} \frac{b_0(p) + b_1(p) z^{-1} + b_2(p) z^{-2}}{1 + a_1(p) z^{-1} + a_2(p) z^{-2}} \tag{2}$$

The form of Eq. (2) suggests cascading second-order IIR filters in series (the output from one stage becoming the input to the next). Therefore this form is referred to as the Cascade form. For good numerical stability, the IIR filter should be implemented as a cascade of second-order, Direct Form II sections. The data flow for a second-order, Direct-Form II section, or bi-quad, is shown in Figure 1. Note that in Direct Form II the delayed samples are neither the input nor the output samples, but are instead the intermediate values $w[n]$.

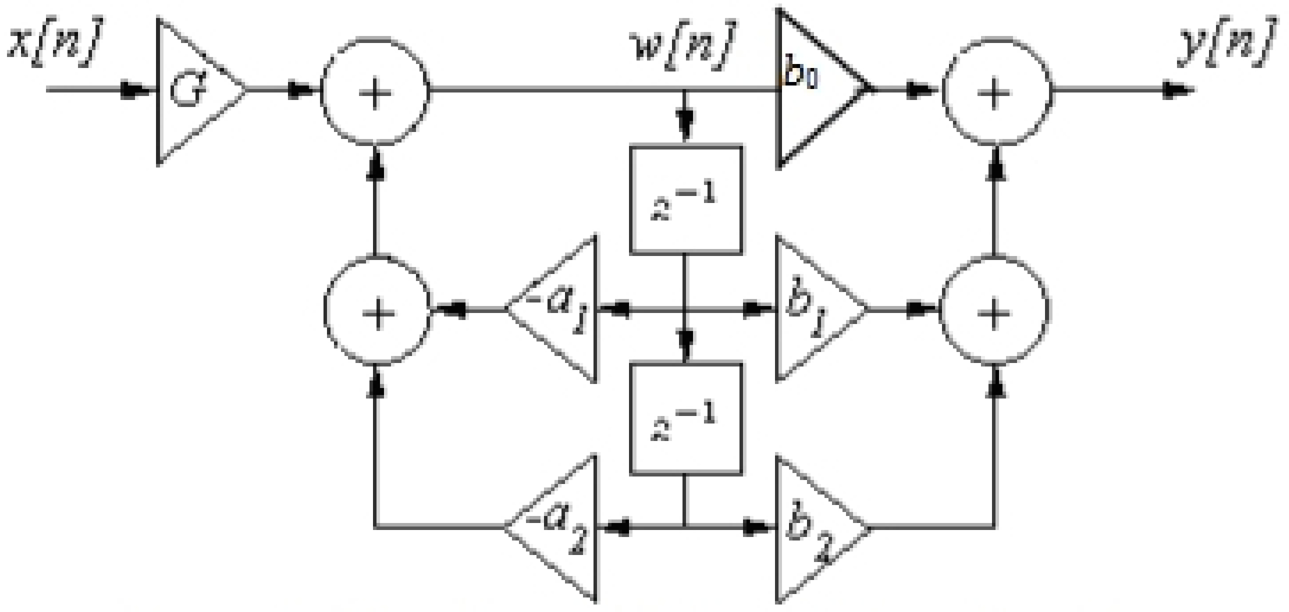

Figure 1: Second-order, Direct Form II section

There are several IIR filter design methods. The main differences between the different methods include the way specifications can be made (i.e. form of spectral magnitude or impulse response) and the type of optimizations used to derive the filter coefficient. The Matlab functions for the main IIR filter designs are Elliptical (*ellip*), Butterworth (*butter*), and Chebyshev (*cheby1* and *cheby2*). The unique features of these approaches will all be examined in the laboratory assignment.

2. Pre-Lab

1. Derive the transfer function of filter shown in Fig. 1. (Hint: obtain transfer function between x and w and then between w and y and use

$$H(z) = \frac{Y(Z)}{X(Z)} = \frac{Y(Z).W(Z)}{W(Z).X(Z)} \qquad (3)$$

   to find the transfer function)

2. (First part is a repeat of Prelab Problem 2, from FIR lab) For a sampling rate of 48kHz design an order-40 low-pass filter having cut-off frequency 10kHz by windowing method. In your design, use Hamming window as the windowing function (see help on *fir1*). Use the *freqz* command to plot the filter's magnitude response (in dB), use the *filter* command to plot the impulse response, and also plot the pole-zeros of the filter in the Z-plane. (New part) Now create a 10th order low-pass Chebyshev Type II digital filter with a stop-band at 11.5kHz with a stop-band attenuation of 55

dB. (a) Plot the IIR filter's magnitude response on the same graph as the previous FIR magnitude response. Compare the graphs and discuss the advantage of the IIR filter in this case. Likewise plot the impulse response for the FIR and IIR filter on the same graph for at least 100 samples. Compare the impulse responses and describe the critical differences between the 2 responses. Finally compare the pole-zero plots of each filter. Plot the roots of the IIR denominator (poles) as red X's and roots of IIR numerator (zeros) as red 0's. One the same graph plot the FIR numerator roots as blue 0's. Compare the 2 patterns and describe how they are both consistent with a low-pass filter.

3. Consider the design the coefficients for a 4th order low-pass elliptical filter with a cutoff of 1000 Hz for a sampling rate of 8000. The passband should have ripple less than .5 dB and the stopband should have ripple less than 10 dB. Verify that this can be done with the following command (see help *ellip*):

   [be4,ae4] = ellip(4,.25,10,.25);

   Now plot the magnitude response obtained with *freqz* to verify conditions are met. Write the transfer function of the filter as a ratio of 4th order polynomials. Then use the *roots* and *poly* commands to write this filter as a product of two 2nd order polynomial ratios. Scale these polynomials so the gain at DC is 1. Draw a block diagram for the implementation using the 4th order polynomials (delays of up to 4 units) and a block diagram for its implementation using two 2nd order systems. You can use the MATLAB command *tf2sos* to check you work, but the script that converts the 4th order system to 2 second order systems must be written using *roots* and *poly*.

3. In-Lab Exercise

These lab exercises focus on the effects of pole placement, robustness of filter implementations, the nature of specifications for the different IIR filters.

1. A pole-zero diagrams are useful for understanding the approximate behavior of a filter. Regions along the unit circle that are close to zeros will result in signal attenuation. The closer the zero is to the unit circle the more dramatic the attenuation. Similarly, regions along the unit circle that are close to poles indicate frequency regions where signal will be amplified. The closer the pole is to the unit circle the more dramatic the amplification. Consider the following transfer function that shows the relationship between a pair of complex conjugate poles (in polar form) and a second-order IIR filter:

$$\begin{aligned} H_i(z) &= \frac{1-r}{(1-re^{j\theta}z^{-1})(1-re^{-j\theta}z^{-1})} \\ &= \frac{1-r}{1-2rcos(\theta)z^{-1}+r^2z^{-2}} \end{aligned} \tag{4}$$