EE 422G - Signals and Systems Laboratory

Lab 9 PID Control

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Objectives:

- Identify the components of a PID controller and derive its impact on the transfer function of a total feedback system.
- Derive state-space models of systems and understand their relationship to transfer functions.
- Simulate a feedback control system in LabVIEW and use it to verify a design of PID controller.

1. Background

This laboratory exercise requires the design of a PID controller for changing the response of a second order plant. LabVIEW software will be used to simulate the performance of the controller and compare its performance with other controllers.

PID stands for Proportional, Integral, and Derivative. Controllers are designed to eliminate the need for continuous operator attention. Examples of common controllers include cruise control for maintaining a constant automobile velocity and a thermostat for maintaining a constant temperature. These controllers automatically adjust a system input (i.e. flow of fuel) based on feedback to hold the measurement to maintain the process at the desired set-point. For a constant reference input (such as speed or temperature), the error is defined as the difference between set-point and measurement:

$$e = R - Y \tag{1}$$

where e is the error, R is the desired set-point, and Y is the measurement of the output being controlled. The variable being adjusted is called the manipulated variable (u) which usually is equal to the output of the controller (or input to the system to be controlled). The relationship between these variables and a feedback control system is illustrated in Fig. 1.

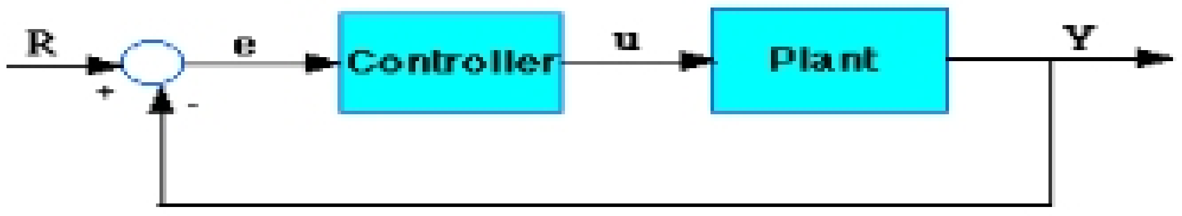

Figure 1. A plant controlled in a feedback system.

The output of PID controllers will change in response to **changes** in the set-point. So for the cruise control example, the measurement *Y* is the speedometer reading, *R* is the value the driver sets through the cruise control unit (desired speed), *E* is the difference between the desired and actual. The controller input, *e*, will dynamically increase and decrease based on other system inputs (such as change incline, weight, wind friction, and driver setting a different velocity value) in addition to known properties of the system, such as the system inertia. The *e* value is processed to dynamically modify the variable *u* (i.e. the flow of fuel or pressure on the break depending on the size, direction, and dynamic of *e*).

The three-term controller

The transfer function of the PID controller can be modeled in the Laplace domain as:

$$K_p + \frac{K_i}{s} + K_d s = \frac{K_d s^2 + K_p s + K_i}{s} \qquad (2)$$

where the following values must be determined to meet the design criteria:

- K_p = Proportional gain
- K_i = Integral gain
- K_d = Derivative gain

Consider how the PID controller works in a closed-loop system using the schematic shown in Fig. 1. The variable *e* represents the tracking error, which is the difference between the desired input value *R* and measured output *Y*. This error signal *e* is the input for the PID controller, and the controller computes both the derivative and the integral of this error signal. The controller output *u* is given by:

$$u = K_p e + K_I \int e dt + K_d \frac{de}{dt} \qquad (3)$$

where the dependence of *e* and *u* on time *t* is implied. The plant accepts input *u* (Fig. 1) which modifies output *Y* and is sent back through the feedback loop to compute the new *e*. This process continues with the objective to drive *e* toward 0.

The characteristics of P, I, and D controllers

While there are several ways to consider control system performance, this laboratory assignment only considers performance metrics associated with a step response or step disturbance. This models the case when the desired value changes or a disturbance pushes the system away from the desired response. The performance metrics in this case describe how well the system returns to the desired behavior after a disturbance or change. These metrics are illustrated in Fig. 2. Three metrics deal with the speed of the response. The **delay time** t_d, is the amount of time it takes the system to achieve 50% of its steady-state value. The definition of **rise time** t_r can vary with the application. Here it is taken as the time for the system response to go between 10% and 90% of the steady-state value. The **settling time** t_s can also vary with the application. In Fig.2 it is defined as the time it takes for the system to remain within 5% of the steady-state response. Lastly, 2 metrics deal with the deviation of the dynamic response from the ideal step response. The steady-state response may not converge on the desired value. In that case a **steady-state** error E_{ss} occurs, which is the difference between the desired and steady-state value of the system response. The **overshoot error** is the maximum deviation resulting from the system overshooting the steady-state response. Note that only under-damped systems will have overshoot. Critically-damped and over-damped systems will not have an overshoot error.

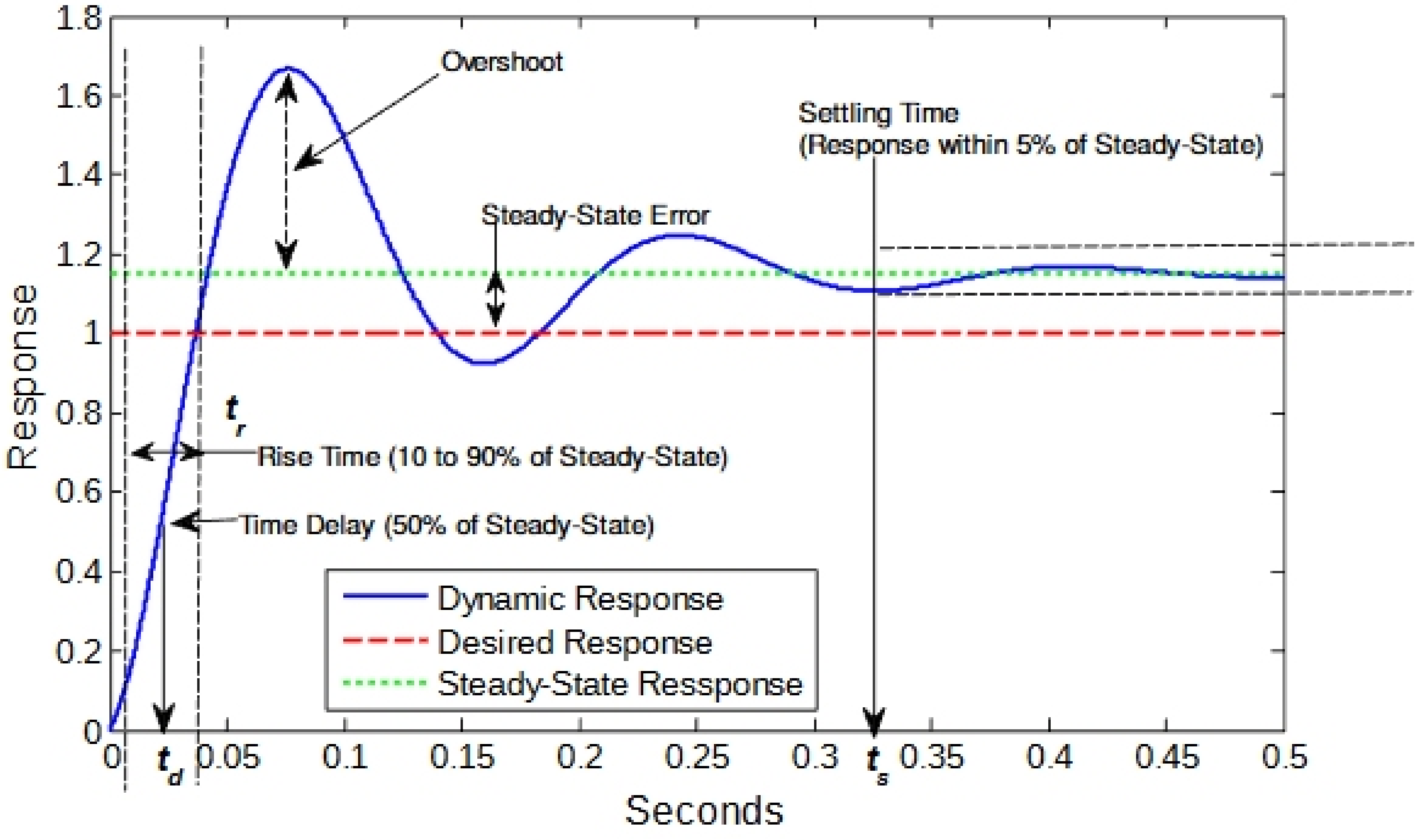

Figure 2. Performance metrics to describe the systems ability to return to or achieve the desired response. These include the time delay t_d, rise time t_r, settling time t_s, and steady-state error E_{ss}.