Introduction to Discrete Structures

Propositional Logic to Informal Proofs

- Statements/propositions
  - Declarative statement that is true or false but not both
  - Examples
    - Sandra De O'Conner was my favorite justice on the Supreme Court (statement/proposition).
    - Read the syllabus carefully (not a statement/proposition).
    - Eat the cake. (not a statement, but a good idea)
    - The cake is a lie (is a statement)
    - X+4=2 (Declarative, not a statement because it could be true or false)
- Propositional variables to represent statements (*p*, *q*, *r*)
  - Ex: *p*: The cake is a lie (*p* = "The cake is a lie")
  - Take on values "true" (*T*) or "false" (*F*)
- Logical Operators
  - Negation (¬) gives the variable the opposite value
    - ¬ *p* = "not *p*" or "the negation of *p*"
  - Conjunction (*p* ˆ *q*)
    - (*p* ˆ *q*) = "*p* and *q*"
    - Compound statement
      - A statement with multiple propositional variables
  - Disjunction (*p* ⋁ *q*)
    - "or" operator
    - At least one variable must be true for (*p* ⋁ *q*) to be true
    - Inclusive
      - Allows both statements to be true
  - XOR Operator (⨁)
    - P ⨁ Q (read "p xor q") is true only when exactly one of P or Q is true
  - Implication (->)
    - P -> Q ("p implies q") = If P=true then Q=true

            - Ex: P= cookies on desk today. Q= cookies will be eaten today.
            - "P is sufficient for Q" or "Q is necessary for P"
            - ¬Q → ¬P is logically equivalent to/is the contrapositive of P → Q (≣ or ⇔)
            - Programming example
                - If (p) {
                  q;
                  }
- Bidirectional Implication (<->)
    - P ⟷ Q (read "P if and only if Q")
    - Abbreviated as "p iff q"
- Definition of implication
    - P -> Q ⇔ ¬P ⋁ Q
- T_0
    - Denotes a tautological statement
- F_0
    - Denotes a counterintuitive statement
- Tautology
    - A compound statement that is always true, no matter the input
- Contradiction
    - A compound statement that is always false, no matter the input
- Contingency
    - A compound statement that is neither tautological nor contradictive
- Converse vs. Inverse
    - Converse of P → Q is Q → P
        - Converse is the contrapositive of the inverse
        - P → Q ⇎ Q → P
    - Inverse of P → Q is ¬P → ¬Q
- Logical connective
    - A logical operator that connects exactly two propositions
- Discrete: exactly X or exactly Y

- Two compound propositions are logically equivalent if the truth values are the same
- Laws of logic
    - Identity laws
        - $P \wedge T_0$ or $P \vee F_0 \Leftrightarrow P$
    - Domination laws
        - $P \vee T_0 \Leftrightarrow T_0$
        - $P \wedge F_0 \Leftrightarrow F_0$
    - Idempotent laws
        - $P \vee P$ or $P \wedge P \Leftrightarrow P$
    - Double Negation law
        - $\neg(\neg P) \Leftrightarrow P$
    - Commutative laws
        - $P \vee Q \Leftrightarrow Q \vee P$
        - $P \wedge Q \Leftrightarrow Q \wedge P$
    - Associative laws
        - $(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$
        - $(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$
    - Distributive laws
        - $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$
        - $P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$
    - DeMorgan's Law
        - $\neg (P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$
        - $\neg (P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$
    - Negation Laws
        - $P \vee \neg P \Leftrightarrow T_0$
        - $P \wedge \neg P \Leftrightarrow F_0$
    - Absorption Laws (really good for making things disappear)
        - $P \vee (P \wedge Q) \Leftrightarrow P$
        - $P \wedge (P \vee Q) \Leftrightarrow P$
- Rule of Substitution