Coulomb's Law

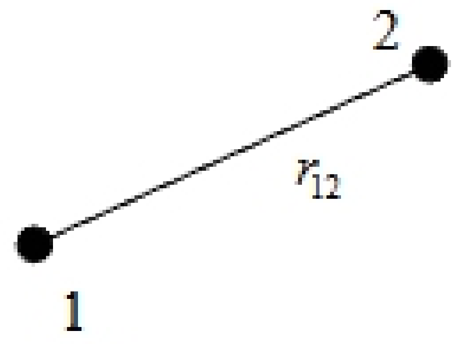
Disclaimer: These lecture notes are not meant to replace the course textbook. The content may be incomplete. Some topics may be unclear. These notes are only meant to be a study aid and a supplement to your own notes. Please report any inaccuracies to the professor.

Electric Charge

- It is an intrinsic property of particles (i.e. electrons and protons)
- Comes in both positive and negative amounts (assignment of + and – chosen by Ben Franklin)
- Usually denote charge by letter "q", unit of measure is the Coulomb, C, in SI units
    - Electron: $q_e = -e = -1.6022\times10^{-19}$ C
    - Proton: $q_p = e = 1.6022\times10^{-19}$ C
    - In fact, all charge is quantized in integer multiples of "e" (see further below)
- Most matter is electrically neutral (balanced: equal amounts + and –)
    - For example, hydrogen, as with all atoms, is neutral. That is lucky for us, otherwise we would have strong attractions to other pieces of matter. But this observation is not explained by any verifiable theory yet!
- Can get a net imbalance of electric charge:
    - Silk on glass ⇒ excess + charge on glass
    - Fur on plastic ⇒ excess – charge on plastic
- Net charge is always conserved
- Like-sign charges repel
- Opposite-sign charges attract
- Need a force law to describe this!

Force Laws

Unit of force is Newtons, N, (kg m s^{-2}), in SI units



Newton's Law of Gravity:

$$\mathbf{F}_{grav} = G\frac{m_1 m_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

m is mass, measured in kg (SI units). Think of it as the "charge" for gravity.

$G = 6.67\times10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$ is Newton's gravitational constant

r_{12} is the distance between two masses, i.e. $|\mathbf{r}_{12}|$

$\hat{\mathbf{r}}_{12}$ is a unit vector pointing along the direction between mass 1 and mass 2

Note that mass is always defined positive (only one type of gravitational "charge"). Also, the force is always attractive, not repulsive. So the direction of the force is always toward another mass.

Coulomb's Law (Law of Electrostatics, 1785):

$$\mathbf{F}_{coul} = K\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

q is electric charge, measured in C (SI units)

$K = \dfrac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{N m}^2\,\text{C}^{-2}$ is the electrostatic constant [Note: some books use "k"]

$\varepsilon_0 = 8.85\times10^{-12}\ \text{C}^2\,\text{N}^{-1}\,\text{m}^{-2}$ is the *electric permittivity* constant

Note that electric charge q can be positive *or* negative. The force is either attractive (opposite charges) or repulsive (like-sign charges). So the direction of the force is either toward another charge (attractive) or oppositely directed from another charge (repulsive). That is, the force is always aligned along $\hat{\mathbf{r}}_{12}$. Use this guidance in determining the direction of a force along a particular axis, not the sign of $q_1 \times q_2$ directly.

Interpretation of force:

A force causes an object to accelerate if it is free to move.

Newton's Second Law: $\mathbf{F} = m\mathbf{a}$

So for the Coulomb force acting on two charged particles otherwise free to move, the acceleration of one of the particles will be:

$$\mathbf{a}_1 = \frac{\mathbf{F}_1}{m_1} = \frac{K}{m_1}\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

Comparison of Gravitational and Electric Forces:

Compare strengths of forces for two objects separated by 1m. Each object has a mass of 1 kg and a charge of 1 C:

$$|F_{grav}| = G\frac{1\cdot1}{1^2} = 6.67\times10^{-11}\ \text{N}$$

$$|F_{coul}| = K\frac{1\cdot1}{1^2} = 9\times10^9\ \text{N} \qquad > 10^{20}\times|F_{grav}|\ !$$

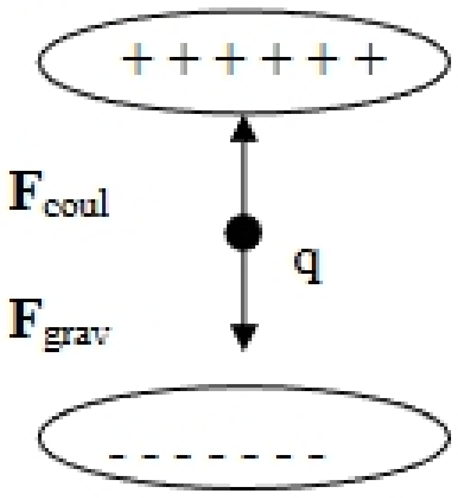
Compare attractive force between electron and proton in hydrogen:

$$|F_{grav}| = G\frac{m_e m_p}{a_0^{\ 2}} = 6.67\times10^{-11}\frac{\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.67\times10^{-27}\ \text{kg}\right)}{\left(0.5\times10^{-10}\ \text{m}\right)^2} = 4\times10^{-47}\ \text{N}$$

$$|F_{coul}| = K\frac{q_e q_p}{a_0^{\ 2}} = \left(9\times10^{9}\ \text{N}\right)\frac{\left(1.6\times10^{-19}\ \text{C}\right)^2}{\left(0.5\times10^{-10}\ \text{m}\right)^2} = 9\times10^{-8}\ \text{N} \quad >10^{40}\times|F_{grav}|\ !$$

Electric Charge Quantization

Experiment done by American physicist Robert Millikan demonstrated that electric charge is quantized.



Millikan's oil drop experiment ⇒
balanced gravitational force, $\mathbf{F}_{grav}$, with electric force, $\mathbf{F}_{coul}$

$\Rightarrow q = n\cdot e \qquad n = 0, \pm1, \pm2, \ldots \qquad e = 1.6022\times10^{-19}\ \text{C}$

There exists an elementary unit of charge!
No smaller charge observed, although "quarks" (constituents of protons and neutrons) are expected to have fractional electric charges. But nevertheless, quantization is still a unique feature. Electrons: $q = -e$, protons: $q = +e$. We don't know why balanced!

1 Coulomb of electrons is $\dfrac{1\ \text{C}}{1.6\times10^{-19}\ \text{C}} = 6\times10^{18}$ electrons!

Electric Charge Conservation

The net sum of electric charge is always conserved. So when a charged conducting object is brought into contact with another conducting object, the charges in the two objects may redistribute, but the net charge of the combined two-object system will remain the same. Likewise, charge is always conserved in reactions:

$$p^+ + e^- \rightarrow {}^1_1H$$
$$e^+ + e^- \rightarrow \gamma\gamma$$
$${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^4_2H$$