multiply by s

$\frac{V_1(s)}{Q_2(s)} = ?$ if $\frac{Q_1(s)}{Q_2(s)}$ is given.

$$V_1(s) = \mathcal{L}[V_1(t)] = \mathcal{L}[\dot{q}_1(t)] = sQ_1(s)$$

$$V_1(s) = sQ_1(s)$$

$$\text{so, } \frac{V_1(s)}{Q_2(s)} = \frac{sQ_1(s)}{Q_2(s)} = s\,G_1(s)$$

$$\frac{V_1(s)}{V_2(s)} = \frac{\cancel{s}Q_1(s)}{\cancel{s}Q_2(s)} \rightarrow \text{same as } \frac{Q_1(s)}{Q_2(s)}$$

State Space

- state vector x is a column vector of n by 1
- For system inputs with n inputs and p outputs
- A state space model

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

- $x[n \text{ by } 1]$, $A[n \text{ by } n]$, $B[n \text{ by } m]$, $y = [p \text{ by } 1]$, $C[p \text{ by } n]$, $D[p \text{ by } m]$

matrix multiplication

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times2} \qquad \underbrace{\underset{(2\times2)(2\times1)}{AB}}_{=2\times1} = \overset{(2\times1)}{C} \qquad \cancel{\underset{(2\times1)(2\times2)}{BA}}$$

$$B = \begin{bmatrix} e \\ f \end{bmatrix}_{2\times1}$$

$$AB = C = \begin{bmatrix} ae + bf \\ ce + df \end{bmatrix}_{2\times1}$$

$$D = \begin{bmatrix} 1 & 7 \\ 3 & 9 \end{bmatrix}$$

$$\underset{2\times2\ \ 2\times2\ \ 2\times2}{AD = E} = \begin{bmatrix} a + 3b & 7a + 9b \\ c + 3d & 7c + 9d \end{bmatrix}$$

$$\underset{n\times 1}{\dot{x}} = \underset{n\times n}{A}\,\underset{n\times 1}{x} + \underset{n\times m}{B}\,\underset{m\times 1}{u}$$

$$\underset{p\times 1}{y} = \underset{p\times n}{C}\,\underset{n\times 1}{x} + \underset{p\times m}{D}\,\underset{m\times 1}{u}$$

$a\dot{x} + bx = u$ — first order

x: is only state variable/vector $n=1$

u: input $m=1$

$y = x$ $p=1$

$a\dot{x} = u - bx \longrightarrow \boxed{\dot{x} = \frac{-b}{a}x + \frac{1}{a}u \quad y = x}$ in most case $D=0$

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = F$$ — 2^{nd} order, so $n=2$

u

input: $m=1$

y

output: $p=2$ (velocity and displacement) or $p=1$

$$\underline{x} = \begin{bmatrix} \text{position } x \\ \text{velocity } \dot{x} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\dot{\underline{x}} = A\underline{x} + Bu$$

$$y = C\underline{x} + Du$$

$$\dot{\underline{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}$$

$$\ddot{x} = \frac{F}{m} - 2\zeta\omega_n\dot{x} - \omega_n^2 x$$

$$\dot{x}_2 = \frac{u}{m} - 2\zeta\omega_n x_2 - \omega_n^2 x_1$$

just rewriting equations in matrix form

$$\dot{x}_1 = x_2 = 0x_1 + 1x_2 + 0u$$

$$y = x_1$$

$$\dot{\underline{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix}}_{A\ [2\times 2]} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}}_{B\ [2\times 1]} u$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C = [1\times 2]} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\emptyset}_{D} u$$