

$$D = 80 + 7 \left| \frac{1}{30} \right.$$

$$W(t) = 40 + 2t$$

$$100 = 40 + 2t$$

$$D = 20 + 30 = 50 \text{ lbs}$$

$$W = 2t$$

$$t = 30$$

Linear Second order differentially Equations

$$y'' + P(x)y' + Q(x)y = G(x)$$

$$y'' + P(x)y' + Q(x)y = 0 \leftarrow \text{Homogeneous}$$

\* If  $P$  and  $Q$  are constants

Then  $f+g$  is also a solution  $\Rightarrow$  CF, CG is a solution

$$f'' + 5f' + 2f + 4g' + 6g = 0$$

Example 1

$$y'' - 8y' + 12y = 0 \quad \text{assume one has } e^{rx}$$

$$y_1 = e^{rx}$$

$$P = e^{2x}, e^{6x}$$

$$y = C_1 e^{2x} + C_2 e^{6x}$$

$$(e^{rx})'' - 8(e^{rx})' + 12e^{rx} = 0$$

$$r^2 e^{rx} - 8r e^{rx} + 12e^{rx} = 0$$

$$e^{rx} (r^2 - 8r + 12) = 0$$

$$(r-2)(r-6)$$

$$r = 2, 6$$

$$y(0) = 3$$

$$y'(0) = 2$$

If  $f$  is a solution

then  $cf$  is also a solution

$$C(f'' + P(x)f' + Q(x)f = 0)$$

$$(cf)' = e^{pf}$$

$$3 = C_1 e^{2(0)} + C_2$$

$$3 = C_1 + C_2$$

$$2 = 2C_1 + 6C_2$$

$$\begin{vmatrix} 3 & 1 \\ 2 & 6 \end{vmatrix}$$

$$C_1 = \frac{\begin{vmatrix} 3 & 1 \\ 2 & 6 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 6 \end{vmatrix}}$$

$$\frac{3(6) - (1)(2)}{6(6) - (1)(2)} = \frac{16}{4} = 4$$

$$C_2 = -1 = \frac{\begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 6 \end{vmatrix}} = \frac{2-6}{6-2} = -1$$

$$2y'' - 8y' - 6y = 0$$

$$y(0) = 2$$

$$y'(0) = 5$$

$$HW = \begin{pmatrix} 1 \\ -13 \\ 17 \end{pmatrix}$$

$$r^2 - 4r - 3 = 0$$

$$r_1 = 2 + \sqrt{7}$$

$$r_2 = 2 - \sqrt{7}$$

$$y = C_1 e^{(2+\sqrt{7})x} + C_2 e^{(2-\sqrt{7})x}$$

$$C_1 = \frac{-4 \pm \sqrt{16-4(-3)}}{2} = -2 \pm \sqrt{7}$$

$$y = (C_1(-2+\sqrt{7})) + C_2(-2-\sqrt{7}) e^{(2+\sqrt{7})x} + C_2(-2-\sqrt{7}) e^{(2-\sqrt{7})x}$$

$$\frac{2(-2-\sqrt{7}) - 5}{(-2-\sqrt{7}) - (-2+\sqrt{7})} = \frac{-4-2\sqrt{7}-5}{-2\sqrt{7}}$$

$$C_1 = \frac{\begin{vmatrix} 2 & 1 \\ -2-\sqrt{7} & 1 \end{vmatrix}}{\begin{vmatrix} -2+\sqrt{7} & 1 \\ -2-\sqrt{7} & 1 \end{vmatrix}}$$

$$C_1 = \frac{-9-2\sqrt{7}}{-2\sqrt{7}}$$

$$C_2 = \frac{\begin{vmatrix} 1 & 2 \\ -2+\sqrt{7} & 5 \end{vmatrix}}{\begin{vmatrix} -2+\sqrt{7} & 1 \\ -2-\sqrt{7} & 1 \end{vmatrix}} = \frac{9-2\sqrt{7}}{-2\sqrt{7}}$$

$$y = \left( \frac{-9-2\sqrt{7}}{-2\sqrt{7}} \right) e^{(2+\sqrt{7})x} + \left( \frac{9-2\sqrt{7}}{-2\sqrt{7}} \right) e^{(2-\sqrt{7})x}$$