

1. Function Domain

- (a) What are the main concerns when finding the domain of a function? Why?
 (b) Find the domain of the following functions and express in interval notation:

i. $g(x) = \frac{x-2}{x^2-4}$ ii. $h(x) = \frac{5x+3}{\sqrt{x^2-8x+16}}$ iii. $f(x) = \frac{1}{x^2+3}$

- (c) And some more (express in set notation):

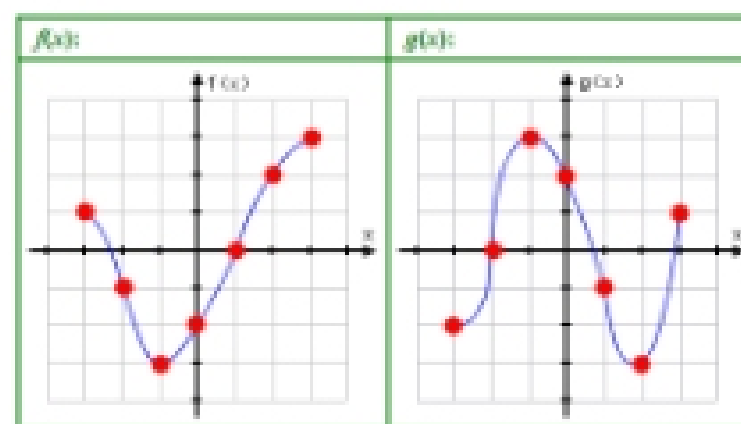
i. $f(x) = \frac{6x^2+2}{\sqrt{x^2-8}}$ ii. $g(x) = \sqrt{\frac{1}{1-|3-x|}}$ iii. $h(x) = \sqrt{(1-x)(-5-x)}$

2. Compositions of functions: For each pair of functions below, simplify $g \circ f(x)$ and find the domain of $g \circ f$.

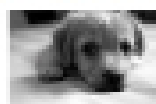
(a) $f = \frac{x}{x-3}$ and $g = \frac{1}{x+2}$

(b) $f = 3 - \sqrt{x+4}$ and $g = \frac{2}{1-\sqrt{x}}$

3. Given the functions f and g to the right, find $g \circ f$ for $x = -3, -2, -1, 0, 1, 2, 3$. Sketch $g \circ f$.



4. WARNING: This problem will require you to think. An alien has kidnapped your



dog. To get your dog back, the alien is requiring you to answer some questions about two functions, f and g . Being a fair alien, the alien provides the following facts:

- $f(4) = 26$
- $g(x) = \frac{\sqrt{4x-100}}{x-2}$
- On the interval $(3, \infty)$, f is defined and increasing.

Now answer the following questions. The alien is not satisfied with just an answer; a correct explanation is necessary to guarantee the safety of your dog.

- (a) What is the domain of g ?
 (b) Make a rough sketch of what f might look like from $(3, \infty)$
 (c) Suppose $h(x) = g \circ f(x)$. Can you guarantee $h(3)$ is defined? Can you guarantee $h(5)$ is defined?

5. Let us talk about sequences. Sequences are **ordered** lists. They can be finite or infinite. What are the next three terms of each of the following sequences, assuming the pattern continues?

(a) 1, 3, 5, 7, ...

(b) -1, 1, -1, 1, ...

(c) 2, 4, 8, 16, ...

(d) 3, 3, 3, 3, ...

(e) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 1, 2, ...

(f) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots$

(g) $\frac{2}{3}, \frac{-5}{8}, \frac{8}{13}, \frac{-11}{18}, \dots$

(h) $\frac{1}{9}, \frac{2}{16}, \frac{3}{25}, \frac{4}{36}, \dots$

6. Sequences are a lot like functions! In fact, they can be thought of as functions that take a natural number (usually we use n) to a value (usually we use a_n). Can you find functions for the sequences above? Use this function to find the 10th term of the sequence. What about the 100th?

7. We use many of the same words to describe sequences as we do functions. What do you think each of the words below means when describing a sequence? Which of the above sequences do these describe? Why or why not? (I know you don't know the answers already. Think about what the words mean for functions and see if you can guess what they mean for sequences. Then ask me if you are unsure)

(a) Increasing

(d) Unbounded

(g) Alternating

(b) Decreasing

(e) Bounded above

(h) Strictly increasing

(c) Bounded

(f) Bounded below

(i) Strictly decreasing

8. We are often interested in the long-run behavior of a sequence. What kinds of behavior do you see in the examples above? (Hint: there are three main types) If the sequence appears to be approaching a single value, what value is it? This type of behavior is called convergent. If it doesn't, it is called divergent. Which of the sequences above are convergent?

9. Discuss with your group what the most important ideas from the last week have been.

An Ending Thought: *The essence of mathematics is not to make simple things complicated, but to make complicated things simple.*

– S. Gudder