MATH 115: Worksheet 6 **September 30, 2014**

1. For each of the following, identify each as Always True, Sometimes True, Never True:

   (a) If $f(x)$ and $g(x)$ are polynomials, then

   i. $f+g$ is a polynomial
   ii. $f-g$ is a polynomial
   iii. fg is a polynomial
   iv. $\frac{f}{g}$ is a polynomial

   (b) A polynomial may have more than one x-intercept

   (c) A polynomial may have more than one y-intercept

   (d) If $g(x)$ is even degree polynomial and $\lim_{x\to\infty} g(x) = \infty$, then $\lim_{x\to-\infty} g(x) = -\infty$

   (e) If $x = \frac{1}{3}$ is a zero of a polynomial, then $(x + \frac{1}{3})$ is a factor

2. Find an example of a non-constant polynomial, $f(x)$ with the following properties or explain why it is impossible:

   (a) Has degree 4 and roots at -1,1, 2

   (b) Has degree 4 and the only real roots are -1,1,2

   (c) Has degree 4 and roots -1,1,2,3,4

   (d) Has integer coefficients and the only root is $\sqrt{2}$

   (e) $\lim_{x\to\infty} f(x) = 4$

   (f) Is not defined for $x \in (-2, 1)$

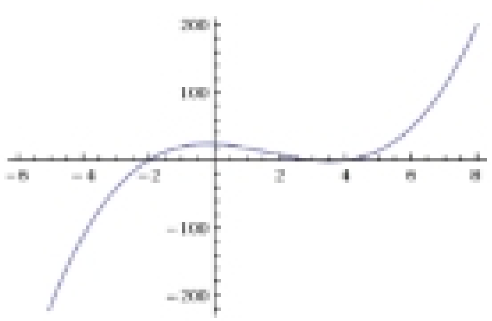
3. Assume the following graphs depict a polynomial, f. Answer the following questions. JUSTIFY YOUR ANSWERS.

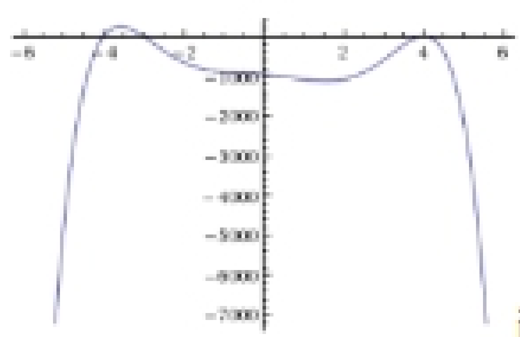
   (a) Is the degree even or odd?

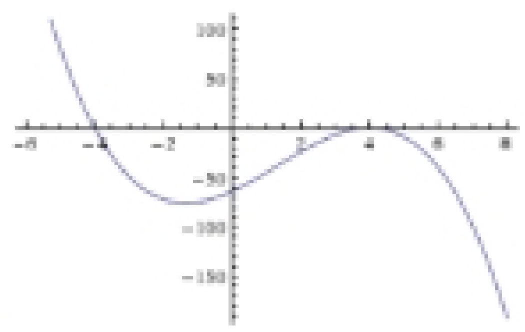
   (b) What is the minimum degree of the polynomial?

   (c) What is the sign of the leading coefficient?

   (d) What is the sign of $f(0)$?

i. 

ii. 

iii. 

4. Which of the following functions are polynomials? If not, EXPLAIN WHY. If so find the leading term, degree, and constant term.

   (a) $\dfrac{2}{x+2}$
   (b) $\sqrt{x}$
   (c) $\sqrt{2}$
   (d) $\frac{4}{7}x^3 - 3x^2 + \pi$
   (e) $x^{-2} + 7x^2 - 11x + 12$
   (f) $(x-3)(x+1)^2(x^2+2)$
   (g) $4^x + x^4$
   (h) $\dfrac{4x+5}{x^{-2}}$

5. For each of the functions above that are polynomials, find the long-term behavior and the y-intercept.

6. For each of the following polynomials, determine the long-term behavior, y-intercepts, x-intercepts, and where each polynomial is positive and negative. Use this to sketch the graph. LABEL all intercepts on your graph. Hint: none of this involves multiplying out the polynomials. Don't.

   (a) $f(x) = (2-x)(2x-8)$
   (b) $g(x) = (1-2x)^2(x+2)$
   (c) $h(x) = (x+2)(x-4)^3(3-x)(1-x)$
   (d) $p(x) = (1-3x)(x^2+4)$

7. Using transformations, graph $f(x) = -2(x-3)^2 + 4$. Does this graph look familiar from the previous question?

8. In addition to knowing their end behavior, in order to graph polynomials with any accuracy, it will also be helpful to have a method for finding their zeros. Fortunately, we have the **Rational Root Theorem:** Given a polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ where $n \geq 1$ and the coefficients a_i are integers, the rational roots of P (if they exist) are of the form $\pm\frac{p}{q}$ where p divides a_0 and q divides a_n.

   (a) Translate this into terms **you** understand.
   (b) Use the Rational Root Theorem to list all the possible rational roots for each of these polynomials.

      i. $f(x) = 4x^3 + 8x^2 - 3x - 9$
      ii. $g(x) = 3x^3 - 14x^2 + 17x - 6$

   (c) For the first polynomial above, $f(x)$, test the possible roots to find which ones are actual roots

9. When a is a root, $P(x)$ has what as a factor? To find what is left, use polynomial long division! Practice with completely factoring (i) above.

10. Let $f(x) = 3x^4 - x^3 - 8x^2 + 2x + 4$

   (a) Use the Rational Root Theorem to find the rational roots of $f(x)$.

   (b) Use the ratinal roots you found to help factor the polynomial. Use your initial factorization to determine all roots of $f(x)$

   (c) Determine the end behavior of $f(x)$

   (d) Use all of the above information to sketch a rough graph of $f(x)$. (It might help to test a point in between each of the roots)

11. Repeat the above steps with the following polynomials:

   (a) $f(x) = x^3 - 2x^2 - 24x$

   (b) $g(x) = -x^3 + 3x - 2$

   (c) $h(x) = x^4 + x^3 - 12x^2 + 4x + 16$

   (d) $p(x) = x^5 - 12x^3 - 16x^2$

An Ending Thought: *You can't fall if you don't climb. But there's no joy in living your whole life on the ground.*

– Unknown