MATH 115: Worksheet 9 — October 21, 2014

Main Concepts

1. Fill out these logarithm rules:
   (a) $\log_b(xy) =$
   (b) $\log_b(\frac{x}{y}) =$
   (c) $\log_b(x^p) =$
2. For the equation $\ln(x) + \ln(x-1) = 1$, which of $x = \frac{1\pm\sqrt{1+4e}}{2}$ can be a solution? Why?
3. A rational function has the form $\frac{p(x)}{q(x)}$. What do we know about $p(x)$ and $q(x)$?
4. What is the domain of a rational function?
5. What conditions do you need for vertical and horizontal asymptotes?

Practice Problems

1. Log rules correspond to the exponent rules:
   (a) Let us first derive the product rule:
      i. Start with $\log_b(x) + \log_b(y) = c$ and raise both sides to the base b power
      ii. Use exponent rules to simplify the left-hand side
      iii. Solve for c.
      iv. Which exponent rule did you use?
   (b) Now let us derive the exponent rule:
      i. Start with $p\log_b(x) = c$ and raise both sides to the base b power
      ii. Use exponent rules to simplify the left-hand side
      iii. Solve for c
      iv. Which exponent rule did you use?
   (c) Use these two rules to derive the quotient rule. Which exponent rules is this similar to?
   (d) We can use these rules to change the base of a logarithm:
      i. Start with $\log_a x = y$ and write as an exponential
      ii. Take the $\log_b$ of both sides
      iii. Solve for y and substitute $\log_a x = y$
      iv. Using this convert $\log_2 x$ to $\log_5$

2. Solve the following equations for x.

(a) $10^{x^2-4} = 1$

(b) $9^x + 3^{x+1} - 18 = 0$

(c) $7^x = 21$

(d) $3^{3x-4} = 15$

(e) $\ln(x^4) - \ln(x^2) = 2$

(f) $\log_3 y + 3\log_3(y^2) = 14$

3. Find an exact value for the following expressions given that $\log_b x = \frac{1}{2}$, $\log_b y = \frac{1}{3}$, and $\log_b z = \frac{1}{5}$.

(a) $\log_b(xz)$ (b) $\log_b\left(\frac{\sqrt{xy}}{z}\right)$ (c) $\log_b\left(\frac{\sqrt{x}}{\sqrt[3]{z}}\right)$ (d) $\log_b\left(\frac{b^2x^{5/2}}{\sqrt{y}}\right)$

4. Suppose $g(x)$ is a rational function that satisfies the following properties:

- $\lim_{x\to\infty} g(x) = 3$
- $\lim_{x\to 2^+} g(x) = -\infty$
- $\lim_{x\to-\infty} g(x) = \infty$
- $\lim_{x\to 2^-} g(x) = -\infty$
- The only intercepts of $g(x)$ are $(5,0)$ and $(0,0)$.

What are the asymptotes of $g(x)$? Sketch a graph (rational functions are continuous on their domain).

5. Let $f(x) = \dfrac{2x+5}{x-1}$.

(a) Find all intercepts of f and where f is not defined?

(b) Find any horizontal asymptotes of f. Remember you can think of $\lim_{x\to\infty} f(x) = \lim_{n\to\infty} f(n)$ and use what you know of sequences.

(c) Consider the limit as f approaches each of the points where f is not defined from the left and the right.

i. First plug in the value. If you get a number, great that is the limit.

ii. If not, does it look like you get $\frac{0}{0}$? If so, try to cancel a common factor.

iii. If not, does it look like $\frac{a}{0}$ where $a \neq 0$? If so, you will get $\pm\infty$. To determine which, look at the sign of the numerator and denominator near your value.

(d) Use this to sketch a graph of f. Plug in additional points if you need to.

6. Repeat the steps above with the following functions

(a) $f(x) = \dfrac{1}{x+1} + 1$ (b) $f(x) = \dfrac{2x^2-8}{x^2+2x}$ (c) $f(x) = \dfrac{x^2-4}{x^2+x-6}$

An Ending Thought: *Enthusiasm moves the world.*

– Arthur Balfour