Improper Fractions.

Two types of improper integrals.

Consider. $$\int \frac{dx}{(x-1)(x-2)}$$

Write

$$\frac{1}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$

$$1 = A(x-2) + B(x-1)$$

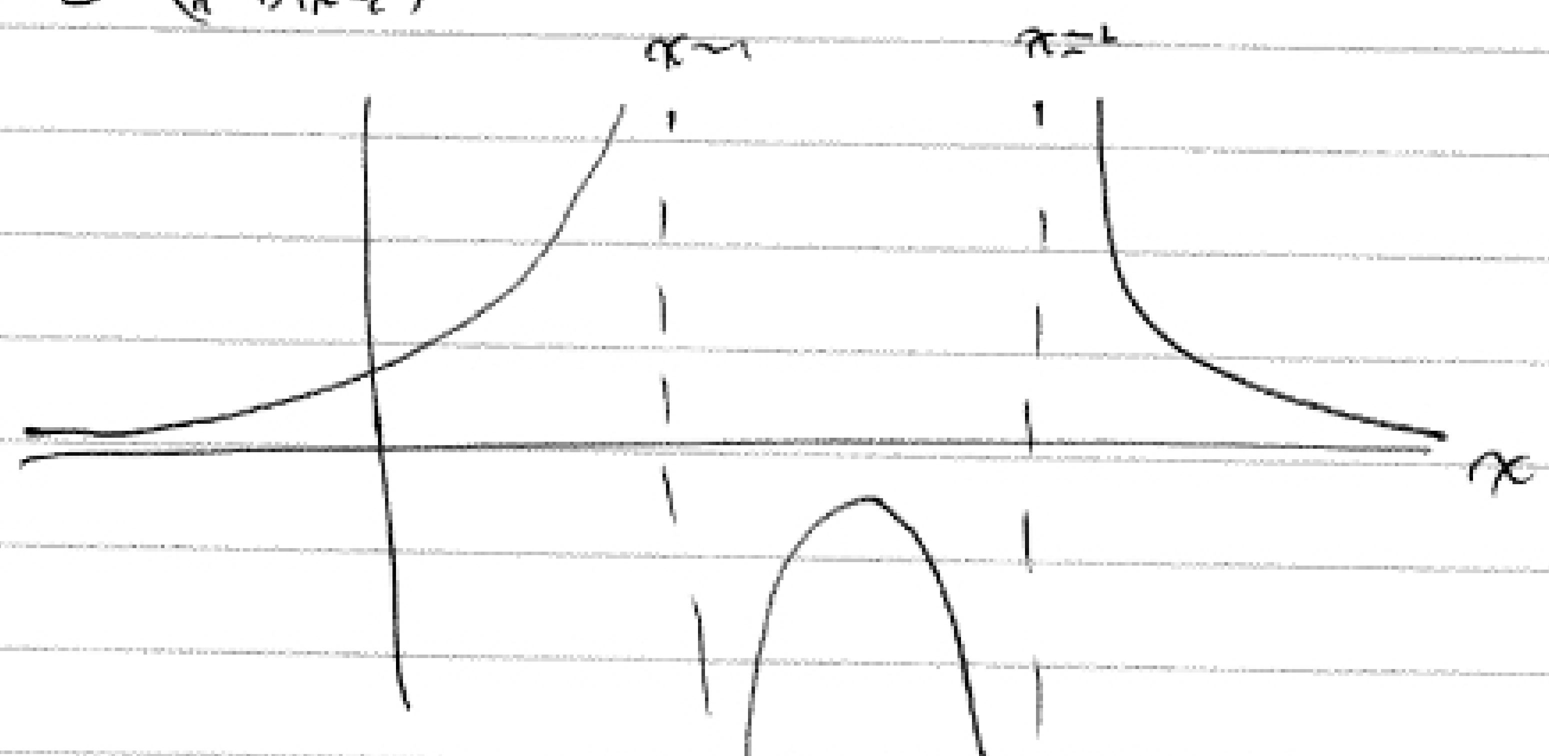
$x=1$: $-A = 1$

$x=2$. $B = 1$

$$\frac{1}{(x-1)(x-2)} = \frac{-1}{x-1} + \frac{1}{x-2}$$

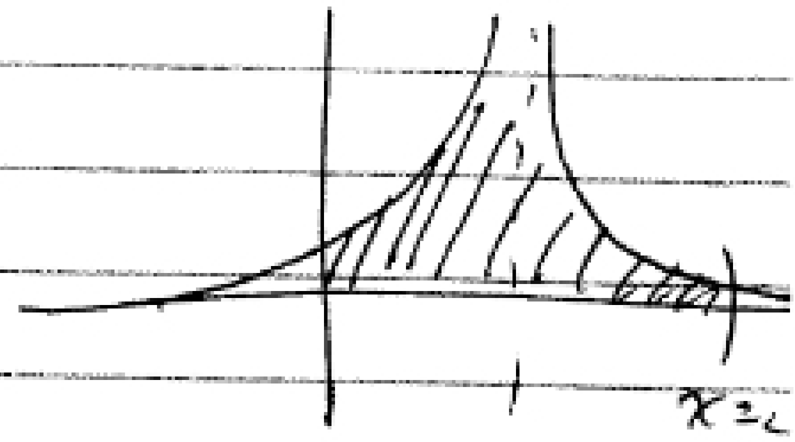
$$\int \frac{dx}{(x-1)(x-2)} = \ln\left|\frac{x-2}{x-1}\right| + C.$$

$$\int_0^3 \frac{dx}{(x-1)(x-2)} = \ln\frac{3}{2} - \ln 2 = \ln 3/4.$$



Q: Does this make sense ??

$$f(x) = \frac{1}{(x-1)^2}$$



$$\int_0^2 \frac{dx}{(x-1)^2} = \left.\frac{-1}{x-1}\right]_{x=0}^{x=2}$$

$$= \frac{-1}{2-1} - \left(\frac{-1}{0-1}\right) = -1 - (1) = -2$$

The area all lies above axis. How can it be negative?

Q: Why does Fundamental Theorem not hold??

A: Function is not continuous on domain.

Type 2 Improper Integral.

[redacted]

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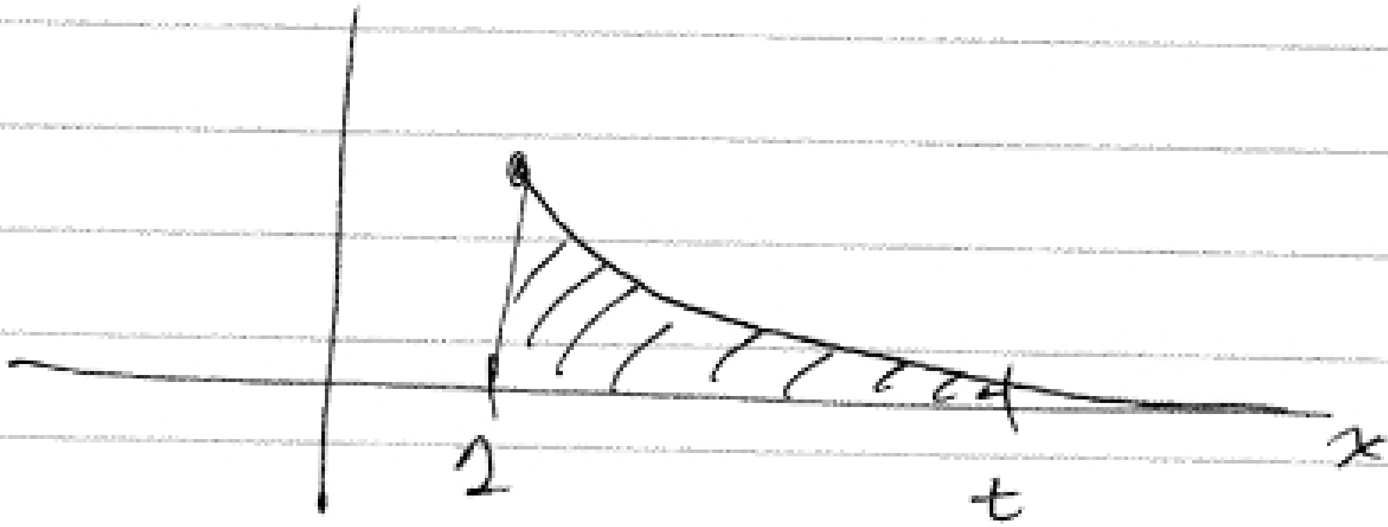
Improper Integral

Q: How do we understand $\int_1^\infty \frac{dx}{x^2}$?



Type 1 Improper Integral.

$$\int_1^t \frac{dx}{x^2} = -\frac{1}{x}\Big]_{x=1}^{x=t} = -\frac{1}{t} - \left(-\frac{1}{1}\right) = 1 - \frac{1}{t}.$$

$$\lim_{t\to\infty} \int_1^t \frac{dx}{x^2} = \lim_{t\to\infty} (1 - 1/t) = 1.$$

Def. We write $\int_a^\infty f(x)\,dx$ as shorthand for

$$\boxed{\lim_{t\to\infty} \int_a^t f(x)\,dx}$$

Similarly, $\int_{-\infty}^a f(x)\,dx = \lim_{t\to-\infty} \int_t^a f(x)\,dx.$

We say convergent if limit exists

divergent if not!

If $\int_a^\infty f(x)dx$, $\int_{-\infty}^a f(x)dx$ are convergent for some a,

$$\int_{-\infty}^\infty f(x)dx = \int_{-\infty}^a f(x)dx + \int_a^\infty f(x)dx.$$

Otherwise, divergent.