Work, Arclength, Surface Area.

- work = force × distance $\qquad W = \int dW = \int F(x)\,dx.$

$dW = F(x)\,dx$

E.g. say it takes 40N to stretch a spring from rest length of 10cm to 15cm.
How much work does it take to go from 15cm to 18cm?

Hooke's Law: $\underset{\text{force}}{F(x)} = \underset{\text{spring constant}}{k} \cdot \underset{\text{distance stretched}}{x}$

$$40\ N = k \cdot 5\ cm$$

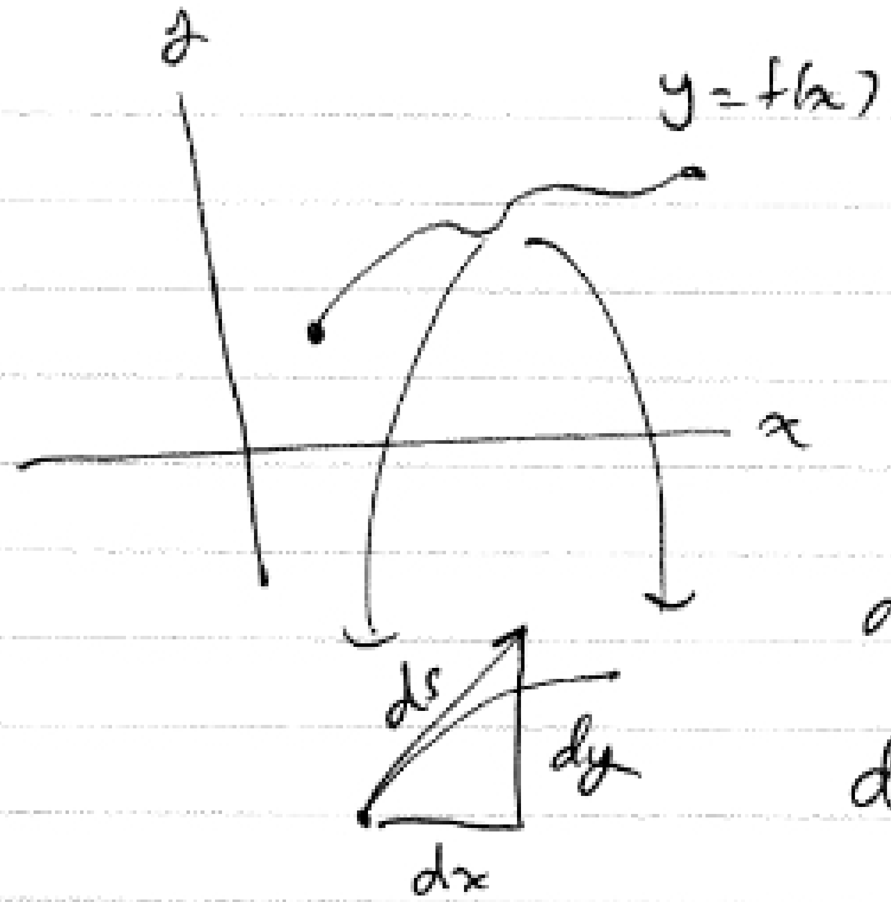
$$k = \frac{40N}{5cm} = \frac{40\ N}{0.05m} = \frac{40\ kg\ m/s^2}{0.05m}$$
$$= 800\ kg/s^2.$$

$$W = \int_{0.05m}^{0.08m} F(x)\,dx = \int_{0.05m}^{0.08m} 800\frac{kg}{s^2}\,x\,dx$$

$$= 400\frac{kg}{s^2}x^2\Big]_{x=0.05m}^{x=0.08m} =$$

$$400\frac{kg}{s^2}\{(0.08m)^2 - (0.05m)^2\} = 1.56\ \frac{kg\,m^2}{s^2} = \underline{1.56\ J}.$$

Arclength. Length of curve?



$$ds^2 = dx^2 + dy^2$$

$$ds = \sqrt{(dx)^2 + (dy)^2}$$

If $y = f(x)$, then we have $dy = f'(x)dx$

$$ds = \sqrt{(dx)^2 + (f'(x)\,dx)^2}$$

$$= \sqrt{1 + f'(x)}\,dx = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

E.g. $y = x^{3/2}$, $1 \le x \le 2$. Length?

$$\frac{dy}{dx} = \frac{3}{2}x^{1/2}, \quad \left(\frac{dy}{dx}\right)^2 = \frac{9}{4}x$$

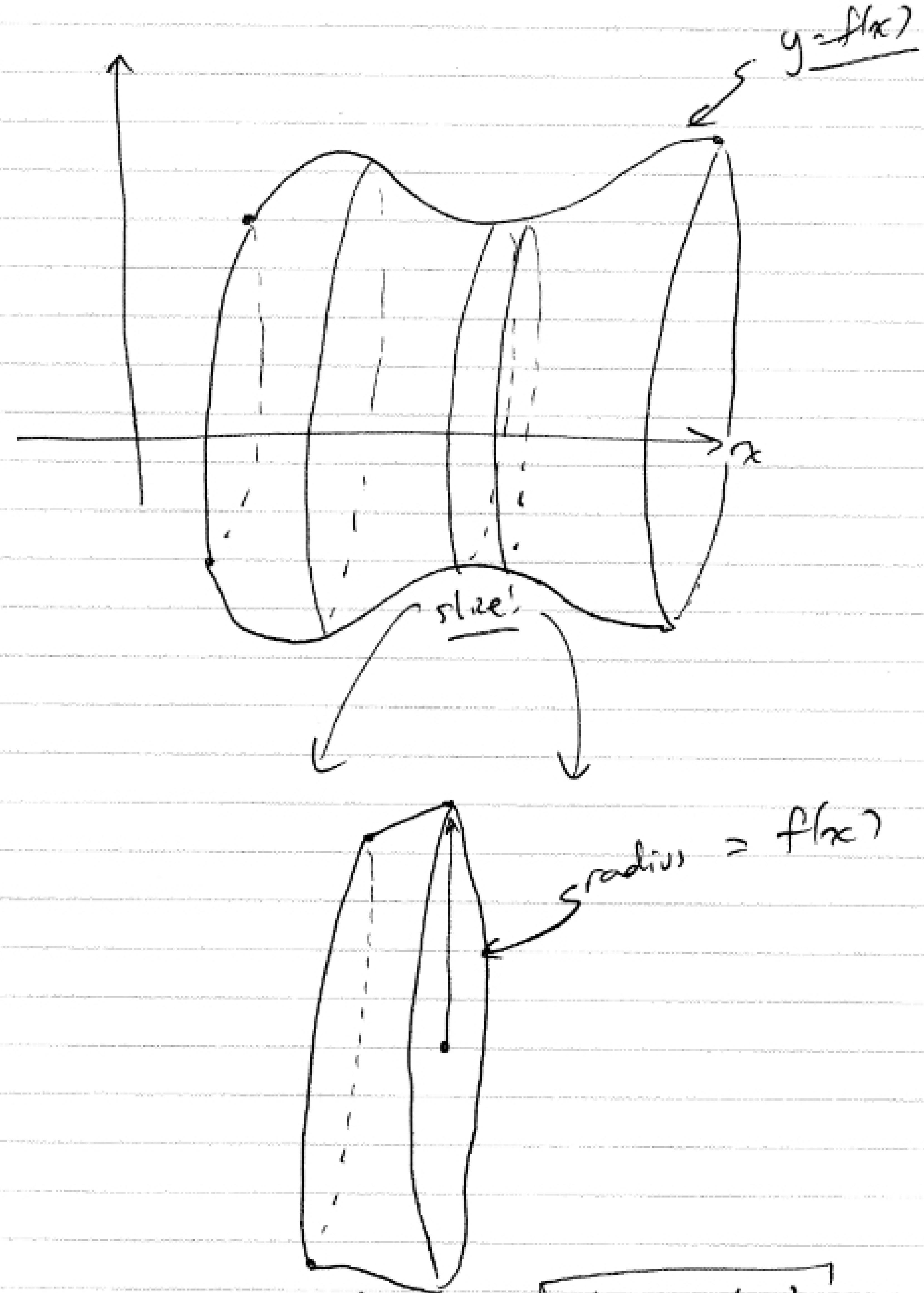
$$\int_1^2 \sqrt{1 + 9/4x}\,dx \qquad \left(1 + \frac{9}{4}x\right)^{3/2} \cdot \frac{2}{3} \cdot \frac{4}{9}$$

$$= \frac{8}{27}\left(1 + 9/4x\right)^{3/2}\Big]_{x=1}^{x=2}$$

$$= \frac{8}{27}\left[\left(\frac{11}{2}\right)^{3/2} - \left(\frac{13}{4}\right)^{3/2}\right] \approx 2.08581$$

Difficulty: arclength gives difficult integrands.

Next question: Consider curve $y = f(x)$.
Rotate around x-axis.
Q: surface area of "shell"?



$$ds = \sqrt{(dx)^2 + (dy)^2}$$