Sequences. Ch. 12.

A sequence is an infinite list of numbers.

$\{a_n = \frac{1}{n}, n = 1, 2, \ldots\}$

$\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{n}, \ldots\}$

We like to find pattern in sequences.

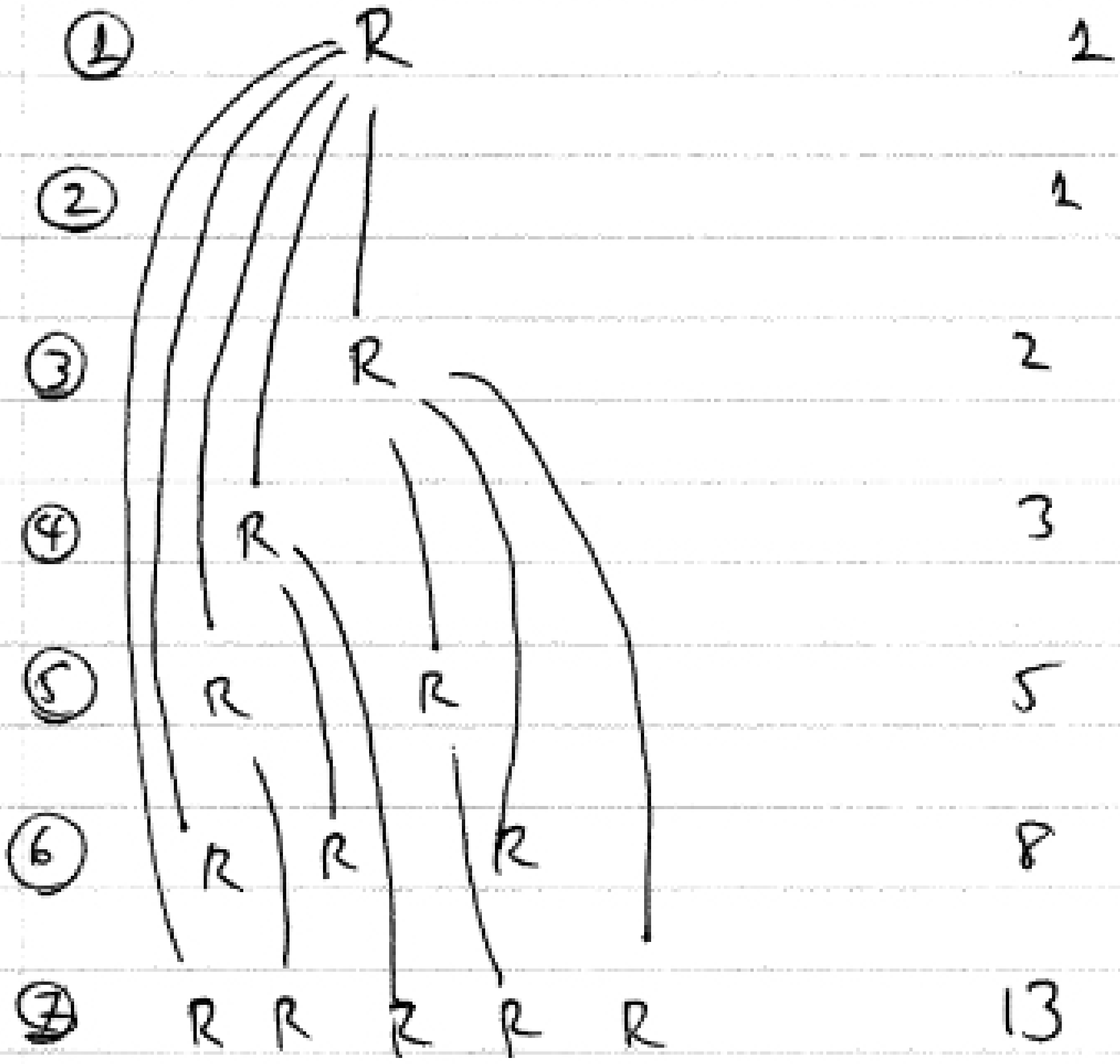
1, 3, 5, 7, 9, (?)

1, 2, 4, 8, 16, (?)

Points.

Old problem. We are given a pair of rabbits.
After one month, give birth to another pair.
Thereafter, each rabbit pair can give birth after two months of age. Rabbits do not die.
Q: How many rabbits do we have in one year?

— Leonardo Pisano Bigollo
Liber Abaci, 1202



Rule: $a_n = a_{n-1} + a_{n-2}$.

Why? Each rabbit that was alive 2 months ago will produce a new rabbit pair now.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144

Leonardo's father was named Bonaccio, so he was called

Filius Bonacci   Remember this we saw in plants?
Fibonacci

We are interested in limits of sequences, given a sequence a_n, we would like to compute

$$\lim_{n\to\infty} a_n.$$

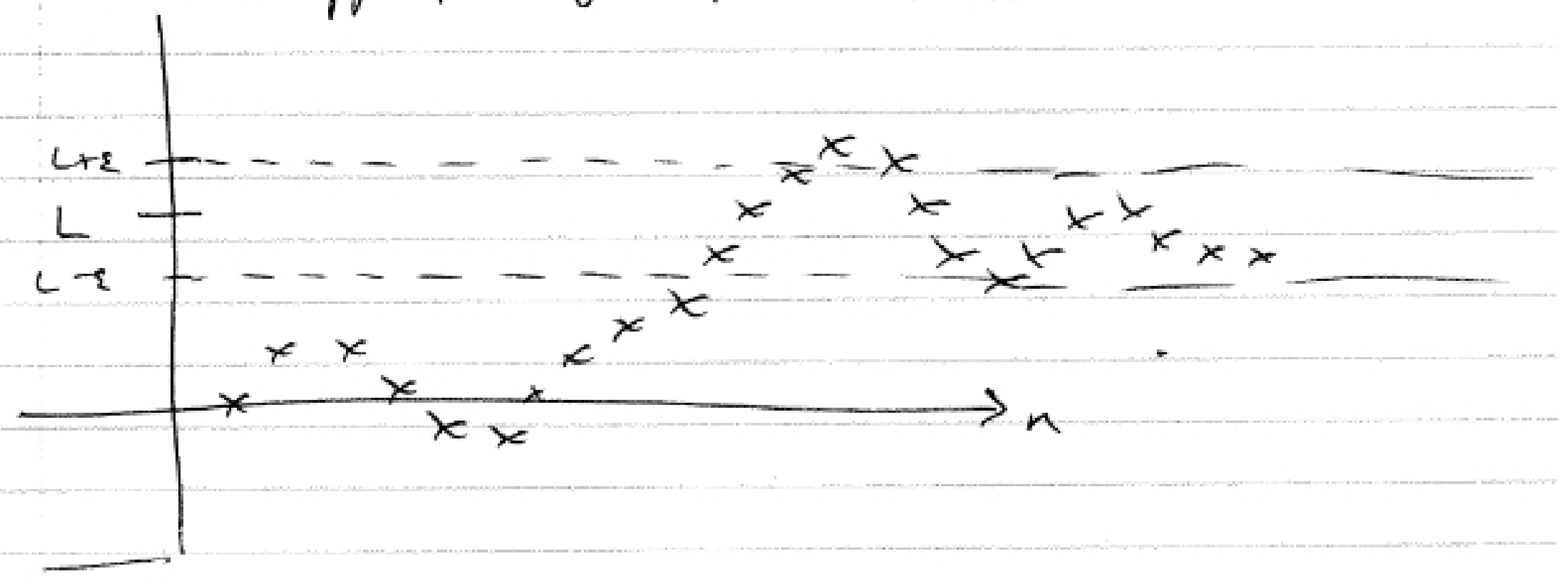
E.g. $a_n = \frac{1}{n}$. It should be that $\lim_{n\to\infty} \frac{1}{n} = 0$.

Def. We say $\lim_{n\to\infty} a_n = L$ to mean:

for any $\varepsilon > 0$, there is $N > 0$ s.t. if $n \geq N$, then

$$|a_n - L| < \varepsilon.$$

If this happens, we say sequence converges.



Theorem. Let $a_n = f(n)$ for some function $f(x)$. If $\lim_{x\to\infty} f(x) = L$, then $\lim_{n\to\infty} a_n = L$.

E.g. $\lim_{n\to\infty} \frac{n}{n^2+4n+1} = 0$, since

$\lim_{x\to\infty} \frac{x}{x^2+4x+1} = 0$ L'H, other limit.