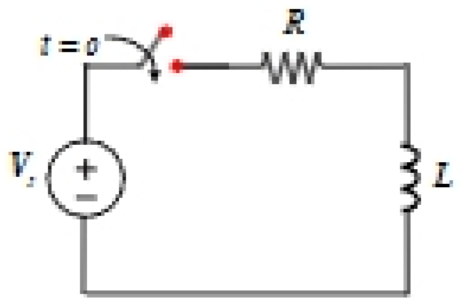
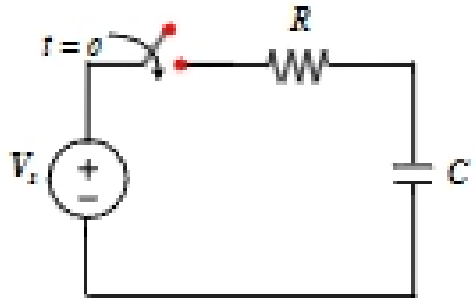
## Step Response of first order circuits

Sudden application or variation of a DC sources.

Initial energy stored within capacitor is defined in terms of initial voltage $v_c$(at t=0)

Initial energy stored within inductor is defined in terms of initial current $i_L$(at t=0)



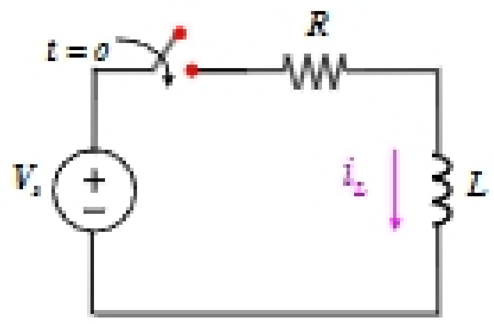




## RL circuit

Switch is closed at t=0

$t = 0^-$ just before action

$t = 0^+$ just after action



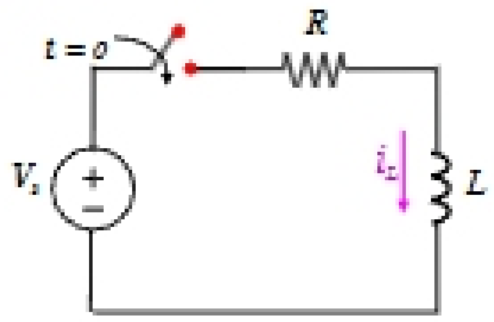

$i_L(0^-) = i_o$

Note!

Inductor current can not change instantaneously

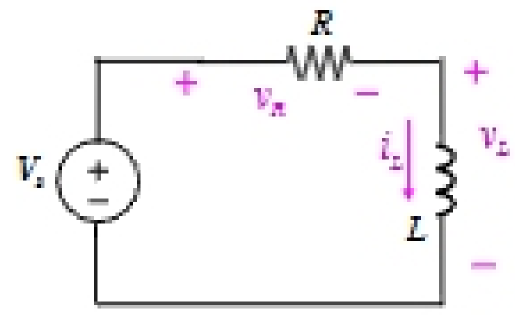
$$i_L(0^-) = i_L(0^-)$$

$t = 0^-$



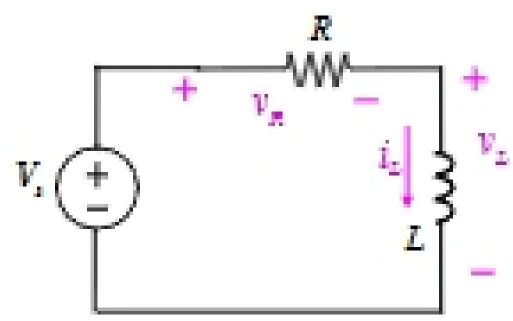

$i_L(0^-) = i_o$

$t = 0^+$




$i_L(0^+) = i_o$

For $t \geq 0^+$



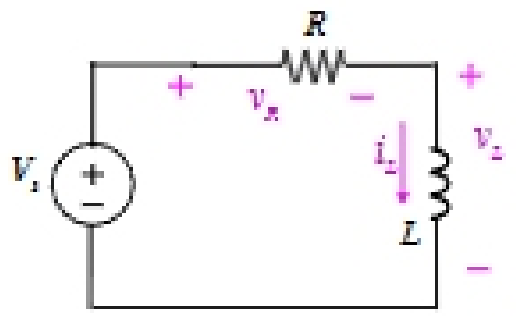

$i_L(0^+) = i_o$

KCL: $i_R = i_L$

KVL: $V_s = v_R + v_L$ $\qquad V_s = i_R R + v_L$ $\qquad V_s = i_L R + L\left(\frac{di_L}{dt}\right)$

$$\frac{di_L}{dt} + \frac{R}{L} i_L = \frac{V_s}{L} \qquad \frac{di_L}{dt} + \frac{1}{\tau} i_L = \frac{V_s}{L} \qquad \tau = \frac{L}{R}$$

First order non-homogeneous D.E.




$i_L(0^+) = i_o$

D.E. $\frac{di_L}{dt} + \frac{1}{\tau} i_L = \frac{V_s}{L}$ where $\tau = \frac{L}{R}$

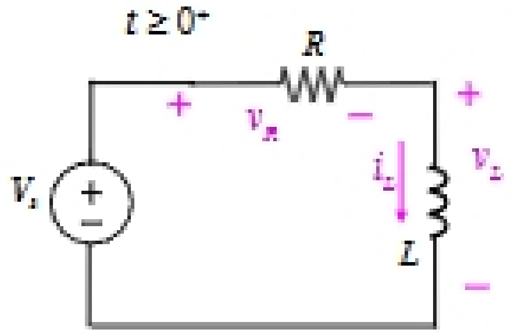
Solution: $i_L(t) = \frac{V_s}{R} + \left(i_o - \frac{V_s}{R}\right) e^{-\frac{t}{\tau}}$ $t \geq 0^+$

Check:

$t = 0^+$ $i_L(0^+) = \frac{V_s}{R} + \left(i_o - \frac{V_s}{R}\right) = i_o$ (initial value)

$t = \infty$ $i_L(\infty) = \frac{V_s}{R} + \left(i_o - \frac{V_s}{R}\right)(0) = \frac{V_s}{R}$ (final value)

$t \geq 0^+$




Summary

D.E. $\frac{di_L}{dt} + \frac{1}{\tau} i_L = \frac{V_s}{L}$

From the circuit connection at $t = 0^+$ $i_L(0^+) = i_o$

From the circuit connection at $t = \infty$ $i_L(\infty) = \frac{V_s}{R}$

From the circuit connection at $t \geq 0^+$ $\tau = \frac{L}{R}$

Solution: $i_L(t) = \frac{V_s}{R} + \left(i_o - \frac{V_s}{R}\right) e^{-\frac{t}{\tau}}$ $t \geq 0^+$

$i_L(t) = (final) + (initial - final)\, e^{-\frac{t}{\tau}}$ $t \geq 0^+$